On the Impact of Forward Markets on Investments in Oligopolistic Markets with Reference to Electricity

Part 2, Uncertain Demand

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Abstract

There is a general agreement since Allaz-Vila's seminal contribution that forward contracts mitigate market power on the spot market. This result is widely quoted and elaborated in studies of restructured power markets where it is generally believed that generators tend to exploit the special characteristics of this industry in order to extract higher prices. Allaz-Vila established their result under the assumption that the production capacities of the players are infinite. This assumption might have applied to the power industry in the early days of restructuring but it no longer holds in today environment of tightening capacity. We show that the Allaz-Vila result no longer holds when capacities are endogenous and constraining generation. Specifically the future market can enhance or mitigate market power when capacities are endogenous and demand is unknown at the time of investment. This result complements Part 1 where the authors show that forward markets do not mitigate market power when capacities are endogenous and demand is known at the time of investment. It also complements other work by Grimm and Zoettl who show that forward markets systematically enhance market power in some symmetric capacity-constrained markets.

1 Introduction

Among the many difficulties encountered in the restructuring of electricity systems, market power and resource adequacy emerge as particularly difficult to handle. The possible exercise of market power and how to mitigate it has retained considerable attention since the California crisis. Many see it as the central cause of the slow progress of electricity restructuring in Europe.[1] Resource adequacy is related to investments levels which have not materialized as initially expected. Both issues are related: insufficient capacity enhances market power and facilitates its exercise. Both issues were treated relatively easily in the past: utilities were regulated at average cost and could generally[2] add capacity costs to their rate bases. Excess investment was even recognized by the literature as a way to increase profits (Averch and Johnson effect (1962)). In restructured markets these problems are much more difficult. While many argue that utilities exercise of market power, this remains difficult to prove in Courts. Notwithstanding the abundance of informal discussions on investment, the literature lacks the sophistication of the capacity expansion models developed for the regulated industry.

This paper analyses a model that combines capacity expansion and a futures market, albeit in an extremely stylized way. The reasoning behind a futures market increasing production is in two parts: generators that have sold part of their supplies forward have less incentive to increase price on the spot market; moreover, a prisoner's dilemma effect identified by Allaz (1992) and Allaz and Vila (1993) induces generators to enter the forward market,

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[1] Difficulties in the restructuring of electricity markets can be attributed to inadequate market architecture (market design) or structure (concentration, inadequate capacities). The question of market architecture has never been tackled seriously in Europe outside of the UK and the Nordic countries.

[2] That is, before the prudence reviews that developped in the US. There was no similar development in Europe.

thereby reducing market power. This argument was developed without considering the effects of capacity limits. With capacities, companies indeed have an additional instrument that has the potential to mitigate the Allaz-Vila effect. In short the question is whether the forward market can still mitigate market power when capacities are endogenous. Conversely, the forward market, because of the Allaz-Vila effect, could induce companies to react by reducing investments as a way of managing spot and forward markets.

We looked at that problem under the simplifying assumption of a fixed demand or single deterministic demand function in Part 1 and came to the following conclusion. The Allaz-Vila effect completely disappears when capacities are endogenous, thereby eliminating the potential of the forward market to reduce market power. Except for possibly destroying the existence of a pure strategy equilibrium, the introduction of a forward market is completely transparent: it does not change the capacity invested and there is no impact on the market power exercised on the spot market. This result has another interesting interpretation. The three-stage game (investment, forward market, spot market) has the same pure strategy equilibrium as a two-stage game (investment, spot market) which is itself equivalent to a single stage game in investment and sales. This result is very much akin to celebrated result established by Kreps and Scheinckman (1983) for a two-stage game (Cournot/Bertrand).

The main result of this paper is less positive: we remove the assumption of a single deterministic demand function and assume that the demand function is unknown at the time of the investment (as it effectively is). We then establish the two following results:

i) the Allaz-Vila result that the forward market mitigates market power no longer holds. In fact the effect of the forward market is ambiguous. It can enhance or mitigate market power and one cannot know which occurs until the model parameters are known;

ii) the equivalence between the multistage and single stage games no longer holds. The solution of the three-stage game (with the forward markets) is different from the solution to the two-stage game (without forward markets).

Our results are developed for the general case without a load curve. Nevertheless, the practical side of these results is the suggestion that we know very little about the behavior of long-term restructured electricity markets when there is market power.

This paper can be related to many areas of the literature. Green (1992) was probably the first one to point the market mitigation effect of forward contracts in restructured electricity systems. His analysis referred to the former England and Wales pool and discussed the impact of exogenously given forward contracts. Many authors later elaborated on the subject, looking at different restructured markets. Models with endogenous contracts were developped after Vila (1992) and Allaz-Vila (1993) seminal contributions. These models are of two-stage games and take different forms. Some adopt the no arbitrage argument that underlies the results of Allaz and Allaz and Vila. This is the case in Green (1999). Some take another view and assume a set of agents maximizing Markowitz type utility functions. Sidiqqui (2003) is an example of these. Last but not least Kamat and Oren (2004) simply question the assumption of no arbitrage and assume that the two-settlement organization offers discrimination possibilities to the generator and hence can enhance market power. While this literature refers to single games, other authors examine the effects of repeated games, specifically Liski and Montero (2006) and Harvey and Hogan (2000). Lastly, Le Coq and Orzin (2002) did experiments testing this result. For a more complete literature review, see part I.

This paper belongs to a smaller stream of the literature. Elaborating on existing economic concepts, e.g. Gabszewicz and Poddar (1997), Murphy and

Smeers (2005) analyze capacity expansion in restructured electricity systems subject to market power. Grimm and Zoettl (2006) further investigate the subject and show that forward markets always have a detrimental effect on investments in some symmetric games as does Adilov (2005).

2 The model

The models analyzed in this paper are constructed as follows.

Suppose two generating companies are in competition, each specializing in one particular technology. This can represent competition between a nuclear generator (e.g. EdF) and a largely coal based utility (e.g. RWE in Germany), or a gas generator (e.g. GdF). Alternatively, both generators can specialize in the same technology. Following much of the economic literature, we assume that there is no existing generation system (see Ehrenmann and Smeers (2005) for a discussion of the impact of that assumption). Each company invests in new capacity and competes on the spot market given its capacity. We thus represent a merchant system. The two models considered in the paper differ in that one has a forward market and the other does not.

In the model with a forward market, the equilibrium in the spot market is found given the capacities and forward positions. The forward-market equilibrium is found given the capacities and taking into account the ensuing spot equilibrium. The players make capacity decisions knowing their impact on the forward and spot equilibria. The model without a forward market has a spot-market equilibrium that is a function of the capacities and the capacity equilibrium is found knowing the effect of the capacity decisions on the spot market.

In contrast to Part 1 we assume that future demand is uncertain. While Part 1 shows that the assumption of a deterministic demand allows one to derive equivalence results similar to those established in Kreps and Scheinkmann, we want to assess the robustness of these results with respect to the very important assumption of a deterministic demand. In reality demand is not known at the time of investment. We model the uncertain demand by assuming an inverted demand function of the form

$$p = \xi - q \tag{1}$$

where p and q respectively denote the price and quantity
ξ is a random intercept with density $f(\xi)$
ξ takes its value in an interval (L, U)

The economic characteristics of the technologies are summarized in the pairs

$$k_i,\ \nu_i \qquad i = 1, 2 \tag{2}$$

where k_i and ν_i are respectively the investment and operating costs of technology i measured in € or \$/Mwh (see Stoft, 2002 for a discussion of these units).

For the sake of technical simplicity and with the view of contributing to the debate on the role of forward markets as a potential instrument for mitigating market power, we assume that the competing companies behave like Cournot players in each of the markets (spot, forward and capacity): they exert market power by setting quantities (energy delivered, forward positions, capacities invested). This is only a working assumption and we make no claim or even suggest that it corresponds to the behavior of a particular company. These quantity variables are denoted

$$x_i, y_i, z_i(\xi) \qquad i = 1, 2 \tag{3}$$

where x_i is the capacity invested by firm i
y_i is the futures position of firm i
$z_i(\xi)$ is the energy delivered by firm i when the demand realization is ξ

Given this background we describe the three markets as follows.

2.1 The spot market

Let x_i and y_i be respectively the capacity and forward positions of agent i when it enters the spot market. We assume that the demand function (1) (that is, the parameter ξ) is revealed after the investments are made and forward positions are taken. For each realization of ξ, the two companies compete as Cournot players on the spot market. Rewriting $z_i(\xi)$ as z_i for the sake of convenience, this implies that each company i takes the production z_{-i} of the other as given and solves

$$\max_{z_i} (\xi - z_i - z_{-i})(z_i - y_i) - \nu_i z_i \tag{4}$$

$$\text{s.t. } 0 \leq z_i \qquad (\omega_i) \tag{5}$$

$$0 \leq x_i - z_i \quad (\lambda_i) \tag{6}$$

This formulation expresses that, after selecting a forward position y_i at an already established forward price, the incentive of the generator to manipulate the market by restricting its generation z_i is limited to the residual market $z_i - y_i$. Let ω_i and λ_i be the dual variables of the constraints $z_i \geq 0$ and $x_i - z_i \geq 0$ respectively. Solving the problems of both generators simultaneously, we obtain the equilibrium conditions of the Cournot spot market.

$$\begin{array}{rcll} 0 & \leq & \xi - 2z_i - z_{-i} - \nu_i + y_i - \lambda_i + \omega_i \perp z_i \geq 0, & i = 1,2 \\ 0 & \leq & x_i - z_i \perp \lambda_i \geq 0, \quad 0 \leq z_i \perp \omega_i \geq 0, & i = 1,2. \end{array} \tag{7}$$

Note that the solution to these conditions is contingent on the realization ξ. The equilibrium on the spot market is thus a parametric complementarity problem. Last we note that the profit accruing to the firm from its operations on the spot market is

$$(\xi - z_i - z_{-i} - \nu_i)(z_i - y_i) \tag{8}$$

where the z_i and z_{-i} satisfy condition (7).

2.2 The forward market

Let y_i be the position taken by agent i on the forward market. We invoke the usual no arbitrage assumption of finance theory which implies that y_i is sold at a price that is the expectation in some risk neutral probability of the spot price (see any textbook of finance, e.g. Hull (2006)). This implies that we reinterpret the distribution $f(\xi)$ of the parameter ξ as a risk neutral probability that develops from the trading of the forwards. The forward price is thus

$$\int_L^U (\xi - z_i - z_{-i}) f(\xi) d\xi. \tag{9}$$

When taking the position y_i given the position y_{-i} of player $-i$, the profit of player i on both the spot and forward markets is then

$$\begin{array}{rl} & y_i \int_L^U (\xi - z_i - z_{-i}) f(\xi) d\xi + \int_L^U (\xi - z_i - z_{-i})(z_i - y_i) f(\xi) d\xi \\ = & \int_L^U (\xi - z_i - z_{-i}) z_i f(\xi) d\xi. \end{array} \tag{10}$$

While this profit does not invoke y_i explicitly it does so implicitly to the extent that the z_i and z_{-i} are parameterized by y_i and y_{-i}. The Cournot problem on the forward market is then defined as follows. Given y_{-i}, generator i solves

$$\max_{y_i} \int_L^U (\xi - z_i - z_{-i}) z_i f(\xi) d\xi \tag{11}$$

where z_i and z_{-i} are the solution of (7).

Equilibrium problems (here equilibrium in the y) subject to equilibrium constraints, here relation (7), (EPEC) belong to the class of Generalized Nash Games (Rosen (1965), Harker (1991)) and suffer from several problems. Specifically, they may or may not have pure strategy equilibria. When pure strategy equilibria exist, they might or might not be unique. When there are multiple equilibria, these solutions can form either a single continuous set or discontinuous sets (Ehrenmann (2004)).

Problem (11) can be converted into a usual Nash equilibrium, albeit without much benefit. Indeed, the solution of the spot equilibrium conditions (7) is unique implying that there exists a unique pair of (nondifferentiable) functions

$$z_i(y_i, y_{-i}; \xi) \qquad \text{and} \qquad z_{-i}(y_i, y_{-i}; \xi) \tag{12}$$

that solves (7). Replacing (12) in (11) we obtain the reformulation of (11)

$$\begin{aligned} & g(x_i, x_{-i}) \\ = \; & \max\nolimits_{y_i} \int_L^U [\xi - z_i(y_i, y_{-i}; \xi) - z_{-i}(y_i, y_{-i}; \xi) - \nu_i] z_i(y_i, y_{-i}; \xi) f(\xi) d\xi \end{aligned} \tag{13}$$

which is a standard Nash equilibrium and no longer an EPEC. Note that this problem is unconstrained in y_i as one assumes that generators can take long or short positions in the futures market and that speculators are ready to take the opposite position. Note also that, in contrast with the spot market, this Nash problem is not guaranteed to have convexity properties with the result that we cannot ascertain that it has a pure strategy solution. The convexity/concavity properties of the second-stage (here spot market) problem of an EPEC are usually lost when moving to the first-stage (here the forward market) of the EPEC. This happens here.

2.3 The capacity market

The profit function of generator i in the forward market depends on the installed capacities x_i, $i = 1, 2$. The Cournot model on the capacity market is obtained by defining the net profit (after accounting for capital charges) of company i

$$p_i(x_i, x_{-i}) = g(x_i, x_{-i}) - k_i x_i \tag{14}$$

with both players simultaneously solving

$$\max_{x_i \geq 0} p_i(x_i, x_{-i}). \tag{15}$$

This is the problem that we are ultimately interested in and for which we want to analyse the impact of a forward market.

3 The standard Allaz-Vila result

Allaz (1992) and Allaz-Vila (1993) showed that the introduction of a forward market mitigates market power. They establish their result in the case of a symmetric equilibrium in a two-stage game where the players optimize their futures position given the resulting equilibrium in the spot market. We first rederive their result in our power market context. We extend Allaz-Vila's model to players operating different technologies and use the model of Section 2 but assume that capacities are non binding whatever the values of ξ. Consider the spot market first. Adapting from Part 1, and assuming $z_i(x_i) > 0$ for all x_i, $i = 1, 2$, the equilibrium conditions on the spot market when the forward positions are respectively y_i and y_{-i} are

$$\xi - 2z_i(\xi) - z_{-i}(\xi) - \nu_i + y_i = 0, \qquad i = 1, 2.$$

This implies

$$z_i(\xi) = \frac{1}{3}[\xi - 2(\nu_i - y_i) + (\nu_{-i} - y_{-i})] \tag{16}$$

and

$$p(\xi) = \frac{1}{3}[\xi + (\nu_i - y_i) + (\nu_{-i} - y_{-i})]. \tag{17}$$

In order to assess the impact of the forward market, we first compute the equilibrium on this market. One can easily verify that the profit of player i for forward positions (y_i, y_{-i}) is

$$\begin{aligned}\Pi_i(y_i, y_{-i}) = \;& \frac{1}{9}\int_L^U (\xi - 2\nu_i + \nu_{-i} - y_i - y_{-i}) \\ & \qquad (\xi - 2\nu_i + \nu_{-i} + 2y_i - y_{-i}) f(\xi) d\xi.\end{aligned}$$

Taking the derivative of Π_i and Π_{-i} with respect to y_i and y_{-i} respectively, we obtain

$$\begin{aligned}\frac{\partial \Pi_i}{\partial y_i} &= \frac{1}{9}\int_L^U (\xi - 2\nu_i + \nu_{-i} - 4y_i - y_{-i}) f(\xi) d\xi = 0 \\ \frac{\partial \Pi_{-i}}{\partial y_{-i}} &= \frac{1}{9}\int_L^U (\xi - \nu_i + 2\nu_{-i} - y_i - 4y_{-i}) f(\xi) d\xi) = 0\end{aligned}$$

$$\frac{\partial^2 \Pi_i}{\partial y_i^2} = -\frac{4}{9} < 0 \qquad \frac{\partial^2 \Pi_{-i}}{\partial y_{-i}^2} = -\frac{4}{9} < 0.$$

The solution of $\frac{\partial \Pi_i}{\partial y_i} = \frac{\partial \Pi_{-i}}{\partial y_{-i}} = 0$ is thus an equilibrium and the corresponding positions on the forward market are given by

$$y_i = \frac{1}{5}[E(\xi) - 3\nu_i + 2\nu_{-i}], \qquad y_{-i} = \frac{1}{5}[E(\xi) - 3\nu_{-i} + 2\nu_i] \tag{18}$$

where

$$E(\xi) = \int_L^U \xi f(\xi) d\xi.$$

Because we assume that z_i is positive for all ξ in the spot market, we have $\xi > \nu_i$, $\forall\ \xi$ and hence

$$y_i + y_{-i} = \frac{1}{5}[(E(\xi) - \nu_i) + (E(\xi) - \nu_{-i})] > 0. \tag{19}$$

Now let $p^f(\xi)$ and $p^0(\xi)$ be respectively the electricity price when there is a forward market and when there is only a spot market. Replacing $y_i + y_{-i}$ by its expression (19), we immediately see from (17) that

$$p^f(\xi) < p^0(\xi).$$

This shows that the forward market decreases the price with respect to the pure spot market. This is the expected Allaz-Vila type result.

4 Back to the capacitated case

The restructuring of electricity began in an environment of relative excess capacity. This is no longer the case today with current concerns about resource adequacy. The introduction of capacities changes the mathematical structure of the Allaz-Vila model. It also changes the results.

4.1 The two capacity models: with and without a forward market

Consider two models consisting of the following components

- *Model with forward market:* this model comprises all modules described in Sections 2.1, 2.2 and 2.3
- *Model without forward market*: this model comprises the different modules described in Sections 2.1, 2.2 and 2.3 where the y_i are set to zero.

The equilibrium model with forward market (15) is mathematically quite complex and beyond the scope of what is generally handled using mathematical programming techniques: it is a three-stage game, a problem more complex than an EPEC. Our objective is not to solve this problem analytically (we do solve numerical examples in Section 6). Instead, we provide sufficient analysis to explore the claim that the forward market always mitigates market power. Our analysis suggests that we know much less about the impact of forward markets than is sometimes thought. A second objective is to investigate the practice that consists of replacing the complex (and currently intractable) three-stage model by the easier (but still complex) two-stage model. These limited objectives justify that we introduce some further simplifying assumptions in the treatment of the spot market as we need them.

4.2 The spot market

As shown in Section 2, the modeling of the spot market drives the rest of the formulation. An equilibrium of the spot market always exists, and under our assumptions it is also unique. This equilibrium can be characterized by specifying the constraints that are binding. The following cases can occur:

$$(i) \qquad 0 < z_i(\xi) < x_i \qquad i = 1,2 \tag{20.1}$$

$$(ii) \qquad 0 < z_i(\xi) < x_i \qquad 0 < z_{-i}(\xi) = x_{-i} \tag{20.2}$$

$$(iii) \qquad 0 < z_i(\xi) = x_i \qquad i = 1,2 \tag{20.3}$$

$$(iv) \qquad 0 = z_i(\xi) \leq x_i \qquad 0 < z_{-i}(\xi) = x_i \tag{20.4}$$

$$(v) \qquad 0 = z_i(\xi) \leq x_i \qquad i = 1, 2. \tag{20.5}$$

For our objective it is sufficient to consider only equilibria for which $z_i > 0$, $i = 1, 2$. This implies that we simplify the complementarity relations (7) into

$$\xi - 2z_i - z_{-i} - \nu_i + y_i + \lambda_i = 0 \quad i = 1, 2 \tag{21.1}$$

$$0 \leq x_i - z_i \perp \lambda_i \geq 0 \qquad i = 1, 2 \tag{21.2}$$

and limit ourselves to the first hree cases in (20).

The set of binding inequalities (20.1), (20.2) and (20.3) depends on the value of ξ. Define $\alpha_i(x, y)$ and $\alpha_{-i}(x, y)$ to be the smallest values of ξ such that

$$\begin{array}{llllll} z_{-i}(\xi) = x_{-i} & \text{and} & z_i(\xi) < x_i & \text{for} & \xi = \alpha_{-i}(x, y) \\ z_{-i}(\xi) = x_{-i} & \text{and} & z_i(\xi) = x_i & \text{for} & \xi = \alpha_i(x, y). \end{array} \tag{22}$$

The definition implies

$$\alpha_{-i}(x, y) < \alpha_i(x, y). \tag{23}$$

The definitions (22) apply in the model without forward markets by setting $y = 0$. Note that one cannot assess ex ante whether $i = 1$ or 2 in (23) solely from the data.

4.2.1 The spot market with forwards positions

We successively consider the first three cases in relations (20).

Case 1. From Part 1, when capacity is not binding

$$z_i^*(y) = \frac{1}{3}[\xi - 2(\nu_i - y_i) + (\nu_{-i} - y_{-i})]. \tag{24}$$

The profit in the spot market is

$$\begin{aligned} & \frac{1}{3}(3\xi - \xi + 2\nu_i - 2y_i - \nu_{-i} + y_{-i} - \xi + 2\nu_{-i} - 2y_{-i} - \nu_i + y_i - 3\nu_i) \\ & \quad \frac{1}{3}[\xi - 2(\nu_i - y_i) + \nu_i - y_{-i}] \\ = \; & \frac{1}{9}(\xi - y_i - y_{-i} - 2\nu_i + \nu_{-i})(\xi - 2\nu_i + 2y_i + \nu_{-i} - y_{-i}). \end{aligned} \tag{25}$$

The profit of player $-i$ is found by interchanging i and $-i$.

Case 2. For $z_{-i} = x_{-i}$ and $z_i < x_i$, we find z_i by solving (21.1) for player i

$$\xi - 2z_i - x_{-i} - \nu_i + y_i = 0$$

or

$$z_i = \frac{\xi - x_{-i} - \nu_i + y_i}{2}. \tag{26}$$

The profit for player i is:

$$\frac{1}{4}(\xi - x_{-i} - \nu_i - y_i)(\xi - x_{-i} - \nu_i + y_i) = \frac{1}{4}[(\xi - x_{-i} - \nu_i)^2 - y_i^2]. \tag{27}$$

The profit for player $-i$ is:

$$\begin{aligned}(\xi - z_i - x_{-i} - \nu_{-i})x_{-i} &= \left(\xi - \frac{\xi - x_{-i} - \nu_i + y_i}{2} - x_{-i} - \nu_{-i}\right) x_{-i} \\ &= \frac{1}{2}(\xi - x_{-i} - 2\nu_{-i} + \nu_i - y_i)x_i.\end{aligned} \tag{28}$$

Case 3. For $z_i = x_i$, $i = 1, 2$ the profit is

$$(\xi - x_i - x_{-i} - \nu_i)(x_i).$$

Next we find the values of α, defined by (22) and (23), where the profit functions switch from Case 1 to Case 2 and from Case 2 to Case 3. These α's become limits of integration for computing the profit accruing to the agents in the forward market. Noting that $\alpha_i(x, y) > \alpha_{-i}(x, y)$ is consistent with Case 2, we can solve for $\alpha_i(x, y)$ and $\alpha_{-i}(x, y)$. Since $\alpha_{-i}(x, y)$ is the point where the solution to the spot market (24) equals capacity, for $-i$, we have

$$x_{-i} = \frac{1}{3}[\xi - 2(\nu_{-i} - y_{-i}) + (\nu_i - y_i)]$$

or

$$\alpha_{-i}(x, y) = 3x_{-i} + 2(\nu_{-i} - y_{-i}) - (\nu - y_i). \tag{29}$$

Similarly,

$$\alpha_i(x, y) = 2x_i + x_{-i} + \nu_i - y_i. \tag{30}$$

4.2.2 The spot market without a forward market

The relevant formulae are obtained by setting $y_i = 0$ in the expressions (25) to (30).

4.3 The forward market

Consider the case where there is a forward market. Using the expressions established in Section 4.2.1, we define the profit function of both agents i and $-i$ (recall that i and $-i$ are identified by the relation $\alpha_{-i}(x,y) < \alpha_i(x,y)$ or $\alpha_{-i}(x) < \alpha_i(x)$). Let $p_i(x,y)$ and $p_{-i}(x,y)$ be the profit functions of generators i and $-i$ respectively,

$$\begin{aligned} p_i(x,y) &= \frac{1}{9}\int_L^{\alpha_{-i}(x,y)} (\xi - y_i - y_{-i} - 2\nu_i + \nu_{-i}) \\ &\qquad (\xi + 2y_i - y_{-i} - 2\nu_i + \nu_{-i}) f(\xi) d\xi \\ &+ \frac{1}{4}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} [(\xi - x_{-i} - \nu_i)^2 - y_i^2)] f(\xi) d\xi \\ &+ \int_{\alpha_i(x,y)}^{U} (\xi - x_i - x_{-i} - \nu_i) x_i f(\xi) d\xi - k_i x_i \end{aligned} \tag{31}$$

$$\begin{aligned} p_{-i}(x,y) &= \frac{1}{9}\int_L^{\alpha_{-i}(x,y)} (\xi - y_i - y_{-i} + \nu_i - 2\nu_{-i}) \\ &\qquad (\xi - y_i + 2y_{-i} + \nu_i - 2\nu_{-i}) f(\xi) d\xi \\ &+ \frac{1}{2}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} (\xi - x_{-i} + \nu_i - 2\nu_{-i} - y_i) x_{-i} f(\xi) d\xi \\ &+ \int_{\alpha_i(x,y)}^{U} (\xi - x_i - x_{-i} - \nu_{-i}) x_{-i} f(\xi) d\xi - k_{-i} x_{-i}. \end{aligned} \tag{32}$$

The equilibrium on the forward market if it exists is obtained by solving

$$\frac{\partial p_i(x,y)}{\partial y_i} = \frac{\partial p_{-i}(x,y)}{\partial y_{-i}} = 0. \tag{33}$$

Existence and uniqueness of the forward equilibrium also require

$$\frac{\partial^2 p_i(x,y)}{\partial y_i^2} < 0 \text{ and} \frac{\partial^2 p_{-i}(x,y)}{\partial y_{-i}^2} < 0.$$

Assuming that the equilibrium exists, these relations define forward positions $y_i(x)$ and $y_{-i}(x)$ for both agents i and $-i$.

4.4 The capacity market

Suppose first that there is a forward market and that its equilibrium exists. The profit function of the capacity market is obtained after replacing the y_i by the equilibrium solution $y(x)$ on the forward market. This can be stated as

$$p_i(x) = p_i[x, y(x)] \qquad i = 1, 2. \tag{34}$$

Consider now the case without a forward market. The objective functions in the capacity game are obtained by setting y_i and y_{-i} to zero in (31) and (32). This leads to

$$\begin{aligned} p_i(x,0) \;&=\; \int_0^{\alpha_{-i}(x)} \frac{1}{9}(\xi - 2\nu_i + \nu_{-i})(\xi - 2\nu_i + \nu_{-i}) f(\xi) d\xi \\ &+ \int_{\alpha_{-i}(x)}^{\alpha_i(x)} \tfrac{1}{4}(\xi - x_{-i} - \nu_i)^2 f(\xi) d\xi \\ &+ \int_{\alpha_i(x)}^{\infty} (\xi - x_i - x_{-i} - \nu_i) x_i f(\xi) d\xi - k_i(x_i) \end{aligned} \tag{35}$$

and

$$\begin{aligned} p_{-i}(x,0) \;&=\; \int_0^{\alpha_{-i}(x)} \frac{1}{9}(\xi - 2\nu_{-i} + \nu_i)(\xi - 2\nu_{-i} + \nu_i) f(\xi) d\xi \\ &+ \int_{\alpha_{-i}(x)}^{\alpha_i(x)} \frac{1}{2}(\xi - x_{-i} - 2\nu_{-i} + \nu_i) x_{-i} f(\xi) d\xi \\ &+ \int_{\alpha_i(x)}^{\infty} (\xi - x_i - x_{-i} - \nu_{-i}) x_{-i} f(\xi) d\xi - k_{-i} x_{-i} \end{aligned} \tag{36}$$

5 Necessary equilibrium conditions

Multistage games do not necessarily have pure strategy equilibria or may have several of them. We analyze this question here by elaborating on the necessary conditions that equilibria should satisfy and discussing why they do not always lead to a pure strategy equilibrium. We assume that the objective functions at each stage are differentiable.

5.1 The necessary conditions of the equilibrium without a forward market

Setting $y_i = y_{-i} = 0$ in relations (29) and (30) we obtain

$$\begin{aligned} \alpha_{-i}(x) &= 3x_{-i} + 2\nu_{-i} - \nu_i && (37) \\ \alpha_i(x) &= 2x_i + x_{-i} + \nu_i. && (38) \end{aligned}$$

The equilibrium conditions are obtained when each agent maximizes its profit by choosing its capacity level or

$$\begin{aligned} \frac{\partial p_i}{\partial x_i} &= \int_{\alpha_i}^{U} (\xi - 2x_i - x_{-i} - \nu_i) f(\xi) d\xi - k_i = 0 && (39) \\ \frac{\partial p_{-i}}{\partial x_{-i}} &= \frac{1}{2} \int_{\alpha_{-i}}^{\alpha_i} (\xi - 2x_{-i} + \nu_i - 2\nu_{-i}) f(\xi) \\ &+ \int_{\alpha_i}^{U} (\xi - x_i - 2x_{-i} - \nu_{-i}) f(\xi) d\xi - k_{-i} = 0. && (40) \end{aligned}$$

Relation (39) can be rewritten as

$$\int_{\alpha_i(x)}^{U} (\xi - \alpha_i) f(\xi) = k_i.$$

It is an equation in α_i from which we infer an equivalent relation

$$\alpha_i(x) = 2x_i + x_{-i} + \nu_i = \overline{\alpha}_i.$$

An equilibrium must satisfy this relation with $\overline{\alpha}_i < U$. The second order condition of (39) is obtained as

$$\frac{\partial^2 p_i}{\partial x_i^2} = \int_{\alpha_i}^{U} (-2) f(\xi) d\xi - (\alpha_i - \alpha_i) \frac{\partial \alpha_i}{\partial x_i} = -2 \int_{\alpha_i}^{U} f(\xi) < 0. \qquad (41)$$

Consider now the second order condition $\frac{\partial^2 p_{-i}}{\partial x_{-i}^2}$. We have

$$\begin{aligned} \frac{\partial^2 p_{-i}}{\partial x_{-i}^2} &= \frac{1}{2} \int_{\alpha_{-i}}^{\alpha_i} (-2) f(\xi) d\xi + \int_{\alpha_i}^{U} (-2) f(\xi) d\xi \\ &+ \frac{1}{2} (\alpha_i - 2x_{-i} + \nu_i - 2\nu_{-i}) f(\alpha_i) \frac{\partial \alpha_i}{\partial x_{-i}} \\ &- \frac{1}{2} (\alpha_{-i} - 2x_{-i} + \nu_i - 2\nu_{-i}) f(\alpha_{-i}) \frac{\partial \alpha_{-i}}{\partial x_{-i}} \\ &- (\alpha_i - x_i - 2x_{-i} - \nu_{-i}) f(\alpha_i) \frac{\partial \alpha_i}{\partial x_{-i}}. \end{aligned}$$

The last three terms can be written after replacement of $\alpha_i, \alpha_{-i}, \frac{\partial \alpha_i}{\partial x_{-i}}$ and $\frac{\partial \alpha_{-i}}{\partial x_{-i}}$ by their values

$$\begin{aligned} & f(\alpha_i)(x_i - \tfrac{x_{-i}}{2} + \nu_i - \nu_{-i}) - \tfrac{3}{2} f(\alpha_{-i}) x_{-i} \\ - \; & f(\alpha_i)(x_i - x_{-i} + \nu_i - \nu_{-i}) \\ = \; & \tfrac{x_{-i}}{2}(f(\alpha_i) - 3f(\alpha_{-i})). \end{aligned}$$

To sum up, we have

$$\begin{aligned} \frac{\partial^2 p_{-i}}{\partial x_{-i}^2} &= -\int_{\alpha_{-i}}^{\alpha_i} f(\xi) d\xi - 2\int_{\alpha_i}^{U} f(\xi) d\xi \\ & \quad -\tfrac{x_{-i}}{2}(3f(\alpha_{-i}) - f(\alpha_i)). \end{aligned} \tag{42}$$

The sign of this expression is generally undetermined. It is always negative in the special case of a uniform exponential distribution of ξ.

5.2 Necessary equilibrium conditions with a forward market

We first consider the equilibrium conditions on the forward market and then turn to the capacity market.

5.2.1 First order conditions on the forward market

Let x be given. The necessary conditions of the futures market are given as

$$\frac{\partial p_i}{\partial y_i} = \frac{\partial p_{-i}}{\partial y_{-i}} = 0 \tag{43}$$

where

$$\begin{aligned} \frac{\partial p_i}{\partial y_i} &= \frac{1}{9}\int_{L}^{\alpha_{-i}(x,y)} (\xi - 4y_i - y_{-i} - 2\nu_i + \nu_{-i}) f(\xi) d\xi) \\ & \quad -\tfrac{y_i}{2}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} f(\xi) d\xi \end{aligned} \tag{44}$$

$$\frac{\partial p_{-i}}{\partial y_{-i}} = -\frac{1}{9}\int_{L}^{\alpha_{-i}(x,y)} (\xi - y_i - 4y_{-i} + \nu_i - 2\nu_{-i}) f(\xi) d\xi. \tag{45}$$

Let

$$\psi_{-i}(\xi, x, y) = \frac{1}{9}(\xi - y_i - 4y_{-i} + \nu_i - 2\nu_{-i}) \text{ for } \xi \in [L, \alpha_{-i}(x,y)] \tag{46}$$

and

$$\psi_i(\xi, x, y) = \begin{cases} \frac{1}{9}(\xi - 4y_i - y_{-i} - 2\nu_i + \nu_{-i}) & \text{for } \xi \in [L, \alpha_{-i}(x,y)] \\ -\frac{y_i}{2} & \text{for } \xi \in [\alpha_{-i}(x,y), \alpha_i(x,y)]. \end{cases} \tag{47}$$

Relation (43) can be restated as

$$\Psi_i(x, y) = \int_L^{\alpha_i(x,y)} \psi_i(\xi, x, y) f(\xi) d\xi = 0. \tag{48}$$

$$\Psi_{-i}(x, y) = \int_L^{\alpha_{-i}(x,y)} \psi_{-i}(\xi, x, y) f(\xi) d\xi = 0. \tag{49}$$

Solving these relations together with

$$\begin{aligned} \alpha_{-i}(x, y) &= 3x_{-i} + 2(\nu_{-i} - y_{-i}) - (\nu_i - y_i) & (50) \\ \alpha_i(x, y) &= 2x_i + x_{-i} + \nu_i - y_i & (51) \end{aligned}$$

gives a candidate equilibrium on the forward market.

One immediately sees that

$$y_i = 0; \qquad \alpha_{-i}(x, y) = L$$

always satisfies relations (48) – (51). We refer to a solution with these properties as a corner solution. A solution satisfying $\alpha_{-i}(x, y) > L$ is termed an interior solution.

We first examine second order conditions for both corner and interior equilibria and complement this analysis by a discussion of the reaction curves of the players. This leads us to conclude that in contrast with the infinite capacity model of Allaz-Vila recalled in Section 3, the equilibrium does not necessarily exist on the forward market. We discuss separately the cases of interior and corner solutions.

Second order conditions

(a) interior solution

First note that

$$\begin{aligned}
\frac{\partial^2 p_i}{\partial y_i^2} &= \frac{\partial \Psi_i(x,y)}{\partial y_i} \\
&= -\frac{4}{9}\int_L^{\alpha_{-i}(x,y)} f(\xi)d\xi + \psi_i(\alpha_{-i},x,y)f(\alpha_{-i})\frac{\partial \alpha_{-i}}{\partial y_i} - \frac{1}{2}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} f(\xi)d\xi \\
&\quad + \frac{y_i}{2}[f(\alpha_{-i})\frac{\partial \alpha_{-i}}{\partial y_i} - f(\alpha_i)\frac{\partial \alpha_i}{\partial y_i}] \\
&= -\frac{4}{9}\int_L^{\alpha_{-i}(x,y)} f(\xi)d\xi + \psi_i(\alpha_{-i},x,y)f(\alpha_{-i}) - \frac{1}{2}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} f(\xi)d\xi \\
&\quad + \frac{y_i}{2}[f(\alpha_{-i}) + f(\alpha_i)].
\end{aligned} \tag{52}$$

The sign of this expression is indeterminate.

We also have

$$\begin{aligned}
\frac{\partial^2 p_{-i}}{\partial y_{-i}^2} &= \frac{\partial \Psi_{-i}}{\partial y_{-i}} = -\frac{4}{9}\int_L^{\alpha_{-i}(x,y)} f(\xi)d\xi - 2\psi_{-i}(\alpha_{-i},x,y)f(\alpha_{-i})\frac{\partial \alpha_{-i}}{\partial y_{-i}} \\
&= -\frac{4}{9}\int_L^{\alpha_{-i}(x,y)} f(\xi)d\xi - 2\psi_{-i}(\alpha_{-i},x,y)f(\alpha_{-i}) < 0
\end{aligned} \tag{53}$$

since $\psi_{-i}(\alpha_{-i},x,y) > 0$, $f(\alpha_{-i}) > 0$, and $f(\xi) > 0$.

We conclude that it is impossible to ascertain a priori that the forward market with capacities has an interior equilibrium.

b) Corner solution

We now turn to the corner solution

$$y_i = 0, \qquad \alpha_{-i}(x,y) = L.$$

We here need to distinguish two cases depending on whether the y variable increases or decreases. We first note that decreasing y_i while keeping y_{-i} fixed decreases $\alpha_{-i}(x,y)$ in formula (50). This is not possible since α_{-i} is already at its lower bound. Similarly, when $\lambda > 0$ in (7) at $x_i = L$, α_i does not change.

This implies that we set $\frac{\partial \alpha_{-i}}{\partial y_i} = 0$ in expression (52) which becomes

$$\frac{\partial^2 p_i}{\partial y_i^2} = -\frac{1}{2}\int_L^{\alpha_i} f(\xi)d\xi + \frac{y_i}{2}f(\alpha_i).$$

The expression is negative at $y_i = 0$ and can only remain negative when y_i decreases. The second order condition is satisfied in this case.

We now examine an increase of y_i. Recall that

$$\begin{aligned} & L = 3x_{-i} + 2(\nu_{-i} - y_{-i}) - \nu_i \\ \text{or} \quad & y_{-i} = \tfrac{1}{2}[3x_{-i} - \nu_i + 2\nu_{-i} - L]. \end{aligned}$$

Replacing y_{-i} by this value in $\psi_i(\alpha_{-i}, x, y)$ we get

$$\psi_i(\alpha_{-i}, x, y) = \frac{1}{9}(\frac{3L}{2} - \frac{3}{2}x_{-i} - \frac{3}{2}\nu_i - 4y_i)$$

and the sign of (52) remains undetermined. In conclusion we cannot ascertain ex ante that a corner solution is a maximand for player i.

Consider now player $-i$ and assume a change of y_{-i} when y_i remains at 0. Increasing y_{-i} should decrease α_{-i} in the middle term of (53), which is not possible. We thus set $\frac{\partial \alpha_{-i}}{\partial y_{-i}} = 0$ in relation (53) and for y_{-i} decreasing get

$$\frac{\partial^2 p_{-i}}{\partial y_{-i}^2} = -\frac{4}{9}\int_0^{\alpha_{-i}(x,y)} f(\xi)d\xi < 0.$$

The second order condition is satisfied here.

Suppose we increase y_{-i} while keeping $y_i = 0$. Replacing in $\psi_{-i}(\alpha_{-i}, x, y)$ we obtain

$$\psi_{-i}(\alpha_{-i}, x, y) = \frac{1}{3}(L - x_{-i} + \nu_i - \nu_{-i}).$$

Again the sign of the final expression cannot be determined. In conclusion the above analysis reveals that in contrast with the case of infinite capacities, there is no guarantee that the forward market has an equilibrium.

A reaction curve analysis

We complete the forward-market analysis by exploring the structure of the reaction curves of the two agents in the forward market. This analysis assumes that the first order conditions suffice to determine the optimal behaviour of an agent given the action of the other, which we have seen is not necessarily the case. We now show that even under these additional assumptions the existence of the equilibrium is not guaranteed.

We first establish the formulas for the reaction curves of player $-i$ and then for player i. Note that $\psi_{-i}(\xi, x, y)$ is linear and increasing in ξ. Since $\Psi_{-i}(x, y) = 0$ at equilibrium, with an interior solution we know that $\psi_i(\alpha_{-i}, x, y) > 0$ and $\psi_{-i}(\alpha_{-i}, x, y) > 0$. Moreover setting $\frac{d\Psi_{-i}(x,y)}{dy_i} = 0$ implies

$$\frac{\partial\Psi_{-i}}{\partial y_{-i}} \times \frac{\partial y_{-i}}{\partial y_i} = -\frac{\partial\Psi_{-i}}{\partial y_i}.$$

We solve for $\frac{\partial y_{-i}}{\partial y_i}$ after finding $\frac{\partial\Psi_{-i}}{\partial y_i}$ and $\frac{\partial\Psi_{-i}}{\partial y_{-i}}$

$$\frac{\partial\Psi_{-i}}{\partial y_i} = -\frac{4}{9}\int_L^{\alpha_{-i}(x,y)} f(\xi)d\xi + \psi_{-i}(\alpha_{-i}, x, y)f(\alpha_{-i}). \tag{54}$$

This is indeterminate in sign.

From (53) and (54) we can write

$$\frac{\partial y_{-i}}{\partial y_i} = \frac{-\frac{4}{9}\displaystyle\int_L^{\alpha_{-i}(x,y)} f(\xi)d\xi + \psi_{-i}(\alpha_{-i}, x, y)f(\alpha_{-i})}{\frac{4}{9}\displaystyle\int_L^{\alpha_{-i}(x,y)} f(\xi)d\xi + 2\psi_{-i}(\alpha_{-i}, x, y)f(\alpha_{-i})}. \tag{55}$$

Since both terms in the denominator are positive and the second term has a coefficient of 2, we can infer

$$\frac{\partial y_{-i}}{\partial y_i} > -1$$

but cannot conclude that $\frac{\partial y_{-i}}{\partial y_i} \leq 0$.

Turning now to the reaction curve of y_i in response to y_{-i}, we can state from (44) at a candidate equilibrium

$$\frac{\partial\Psi_i(x,y)}{\partial y_{-i}} = -\frac{1}{9}\int_L^{\alpha_{-i}(x,y)} f(\xi)d\xi - 2\psi_i(\alpha_{-i}, x, y)f(\alpha_{-i}) - y_i f(\alpha_{-i}) < 0.$$

Since

$$\frac{\partial\Psi_i}{\partial y_i} \times \frac{\partial y_i}{\partial y_{-i}} = -\frac{\partial\Psi_i}{\partial y_{-i}}$$

we get

$$\frac{\partial y_i}{\partial y_{-i}} = -\frac{+\displaystyle\int_L^{\alpha_{-i}} f(\xi)d\xi + 2\psi_i(\alpha_{-i},x,y)f(\alpha_{-i}) + y_i\,f(\alpha_{-i})}{[4\displaystyle\int_L^{\alpha_{-i}} f(\xi)d\xi + \psi(\alpha_{-i},x,y)f(\alpha_{-i})] + \frac{1}{2}\int_{\alpha_{-i}}^{\alpha_i} f(\xi)d\xi - \frac{y_i}{2}[f(\alpha_{-i}) + f(\alpha_i)]} \tag{56}$$

from which we cannot derive any properties.

From (55) and (56) we see that the slopes of the reaction function do not necessarily fall in the range of $(-1, 0)$ that we want to see in a game. The same relations illustrate how the properties for an equilibrium hold in the standard Allaz-Vila case where capacities are infinite. In these equations, if we let $\alpha = \infty$, and set to zero all terms except the integrals from L to ∞, we have the reaction functions with infinite capacity, that is reaction functions with no capacity game. In this case the slopes then fall in the range of $(-1, 0)$ and the game of the forward market is well behaved.

This can be illustrated graphically. Plotting ψ_i and ψ_{-i} in (46) and (47), we can see the marginal contribution to profit at each ξ. We can perturb the variables to get a sense of how the profit forward game changes. We begin with ψ_{-i}.

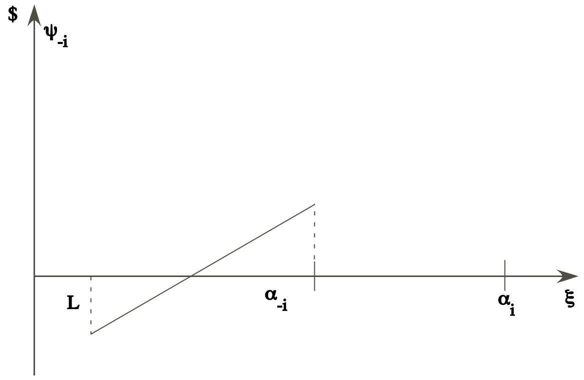

Figure 1: Marginal contribution in the spot market of y_{-i} as a function of ξ at the equilibrium solution as seen in the forward market game

In Figure 1 as ξ increases, the contribution to profit increases linearly and then the contribution stops once capacity is reached, when z_{-i} is equal to x_{-i}. Without a capacity constraint the line would continue indefinitely. We now look at the effect of increasing y_i on λ_{-i}.

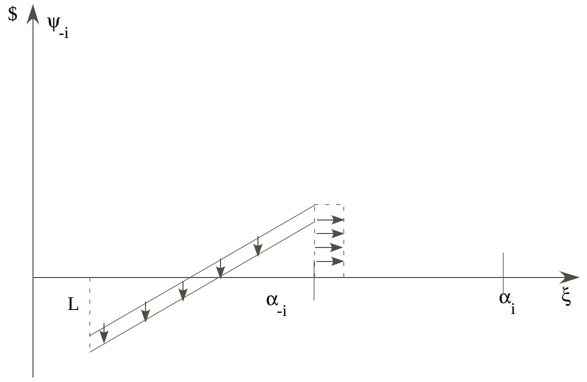

Figure 2: The effect of increasing y_i on λ_{-i}

Increasing y_i for $\xi < \alpha_{-i}$ decreases ψ_{-i}. We also have to take account of the effect on α_{-i}. Since α_{-i} is increasing, the direction in the change in profit is dependent on which area is larger, the decreasing area ranging over the ξ or the increasing area associated with the change in α_{-i}. This cannot be ascertained ex ante and hence the outcome is ambiguous.

Plotting ψ_i we get Figure 3.

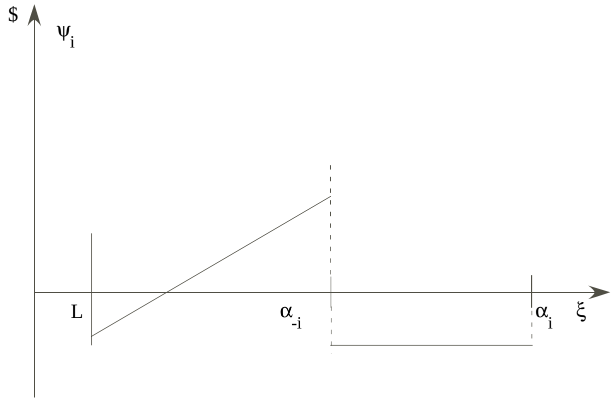

Figure 3: Marginal contribution in the spot market of y_i, ψ_i, as a function of ξ at the equilibrium solution as seen in the forward market game

Note that between the α's the contribution is negative because of the second integral in (44), unlike Figure 1. Increasing x_i enlarges α_i and hence adds to the negative area. The impact of an increase in y_{-i} can be seen in Figure 4.

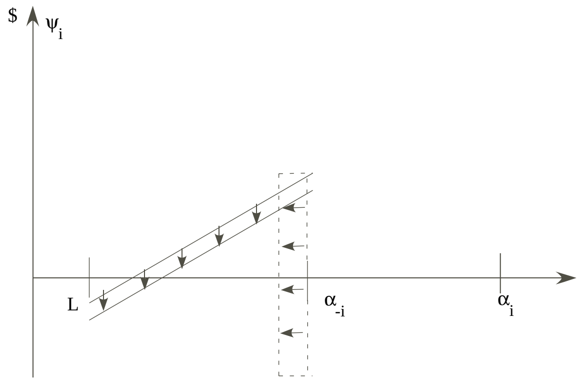

Figure 4: Effect of an increase in y_{-i} on the marginal contribution of y_i, λ_i

Increasing y_{-i} decreases ψ_{-i} in $[L, \alpha_{-i}(x, y)]$ and decreases $\alpha_{-i}(x, y)$. It does not modify $\alpha_i(x, y)$. We see that the effect is unambiguous in that the marginal contribution decreases. This implies that player i sees its marginal profit becoming negative as a result of an increase of y_i. It reacts by decreasing y_{-i}. We are however unable to determine by how much. Again, note that in the forward market game without capacity limits, the negative ψ's between the α's do not exist.

These graphs show that the boundaries of the integrals, the α's, totally change the character of the results of the forward market and create the possibility for capacity to increase or decrease through the addition of a futures market. They also illustrate why our results are different from Adilov (2005). He represents demand uncertainty with the binomial distribution while we use a continuous distribution. In this case, the changes in the vertical segments defined by the boundary of the integrals at α_{-i} that we have drawn in Figures 2 and 3 do not exist because the probabilities used are discrete. Thus, the effect of changing the u's is unambiguous.

5.3 Necessary equilibrium conditions on the capacity market

Assume in the following that the forward market has a unique equilibrium and let $y(x)$ be the corresponding futures positions of the players. We want to explore whether there is an equilibrium on the capacity market.

Let $p_i[x, y(x)]$ be the profit accruing to generator i on the capacity market after taking the optimal forward position $y(x)$. The equilibrium on the capacity market must satisfy

$$\frac{dp_i}{dx_i} = 0 \tag{57}$$

or

$$\frac{\partial p_i}{\partial x_i} + \frac{\partial p_i}{\partial y_i}\frac{\partial y_i}{\partial x_i} + \frac{\partial p_i}{\partial y_{-i}}\frac{\partial y_{-i}}{\partial x_i} = 0. \tag{58}$$

Taking into account that $\frac{\partial p_i}{\partial y_i} = 0$ at the equilibrium on the forward market, we obtain

$$\frac{\partial p_i}{\partial x_i} + \frac{\partial p_i}{\partial y_{-i}}\frac{\partial y_i}{\partial x_i} = 0. \tag{59}$$

Similarly $\frac{dp_{-i}}{dx_{-i}} = 0$ implies

$$\frac{\partial p_{-i}}{\partial x_{-i}} + \frac{\partial p_{-i}}{\partial y_i}\frac{\partial y_i}{\partial x_{-i}} = 0. \tag{60}$$

The establishment of the necessary equilibrium conditions on the capacity market therefore requires computing

(i) $\frac{\partial p_i}{\partial x_i}$ and $\frac{\partial p_{-i}}{\partial x_{-i}}$

(ii) $\frac{\partial p_i}{\partial y_{-i}}$ and $\frac{\partial p_{-i}}{\partial y_i}$

(iii) $\frac{\partial y_i}{\partial x_{-i}}$ and $\frac{\partial y_{-i}}{\partial x_i}$.

We here discuss the equilibrium on the capacity market when the assumed equilibrium on the forward market is a corner solution. The formula for the interior solution are given in Appendix A1.

5.3.1 The forward market has a corner equilibrium

Recall that this equilibrium is characterized by

$$y_i = 0; \qquad y_{-i} \geq \frac{1}{2}[3x_{-i} - \nu_i + 2\nu_{-i} - L].$$

We immediately obtain

$$\frac{\partial y_i}{\partial x_{-i}} = 0; \qquad \frac{\partial y_{-i}}{\partial x_i} = 0.$$

The necessary equilibrium conditions on the capacity market reduce to

$$\frac{\partial p_i}{\partial x_i} = \frac{\partial p_{-i}}{\partial x_{-i}} = 0 \tag{61}$$

which look similar to the equilibrium solutions obtained when there is no forward market. The equilibrium is not the same though, because player $-i$ has a non-zero position on the forward market. The equilibrium condition for player i can be stated as

$$\int_{\alpha_i}^{U} (\xi - 2x_i - x_{-i} - \nu_i) f(\xi) d\xi - k_i = 0 \tag{62}$$

where

$$\alpha_i = 2x_i + x_{-i} + \nu_i \tag{63}$$

since $y_i = 0$. These conditions are again equivalent to

$$\alpha_i(x) = 2x_i + x_{-i} + \nu_i = \overline{\alpha}_i \tag{64}$$

where $\overline{\alpha}_i$ is a solution of

$$\int_{\alpha}^{U} (\xi - \alpha) f(\xi) = k_i$$

that must satisfy $\overline{\alpha}_i \leq U$. This equilibrium condition is thus identical to the one obtained when there is no forward market.

The equilibrium conditions of x_{-i} are different and are

$$\begin{aligned} &\frac{1}{2}\int_{L}^{\alpha_i} (\xi - 2x_{-i} + \nu_i - 2\nu_{-i}) f(\xi) d\xi \\ + &\int_{\alpha_i}^{U} (\xi - x_i - 2x_{-i} - \nu_{-i}) f(\xi) d\xi - k_{-i} = 0 \end{aligned} \tag{65}$$

or

$$\begin{aligned} & \frac{1}{2}\int_L^{\overline{\alpha}_i} \xi f(\xi) d\xi + \int_{\overline{\alpha}_i}^U \xi f(\xi) d\xi \\ = \; & k_{-i} + \frac{1}{2}(-2x_{-i} + \nu_i - 2\nu_{-i})(\overline{\alpha}_i - L) \\ & -(x_i + 2x_{-i} - \nu_{-i})(U - \overline{\alpha}_{-i}) \end{aligned} \tag{66}$$

which, because $\overline{\alpha}_i$ is known, is a linear expression in x_i and x_{-i}. The candidate capacity equilibrium for a corner equilibrium on the forward market is thus found by solving a linear system of equations.

We now verify the second order conditions

$$\begin{aligned} \frac{\partial^2 p_i}{\partial x_i^2} &= -2\int_{\alpha_i}^U f(\xi) d\xi < 0 \\ \frac{\partial^2 p_{-i}}{\partial x_{-i}^2} &= -\frac{1}{2}2\int_L^{\alpha_i} f(\xi) + \frac{1}{2}(\alpha_i - 2x_{-i} + \nu_i - 2\nu_{-i}) f(\alpha) \\ &\quad -2\int_{\alpha_i}^U f(\xi) d\xi - (\alpha_i - x_i - 2x_{-i} - \nu_{-i}) f(\alpha) \\ &= -\int_L^{\alpha_i} f(\xi) d\xi - 2\int_{\alpha}^U f(\xi) + \left(-\frac{\alpha_i}{2} + x_i + x_{-i} + \frac{\nu_i}{2}\right) f(\alpha_i) \\ &= -\int_L^{\alpha_i} f(\xi) d\xi - 2\int_{\alpha_i}^U f(\xi) d\xi + \frac{x_i}{2} f(\alpha_i) \end{aligned} \tag{67}$$

which is again of undeterminate sign. As with the forward market, it is impossible to ascertain ex ante the existence of an equilibrium solution.

5.3.2 Using the reaction functions in the capacity game

We examine the qualitative properties of how the capacities change with the addition of a forward market. We use the reaction fucntions in the capacity game to examine how the solution changes from the equilibrium without a futures market. From (63) given the capacity x_{-i} from the model without a forward market, x_i from this same model is the optimal capacity in the model with a forward market as $y_i = 0$. The left side of (65) evaluated with the capacities set at the levels from the model without a forward market tells us the direction of change in the capacity of player $-i$. If this expression is positive (negative), then the capacity x_{-i} will increase (decrease) and the effect of a change in x_{-i} on x_i will determine the total change in capacity.

Note that the equilibrium condition of the capacity market in the game with no forward market, (40), differs from (66) only in the lower limit of the first integral, α_{-i} versus L. Substracting (40) from the left side of (66), leads to the following,

$$\frac{\partial p_{-i}}{\partial x_{-i}} = \frac{1}{2}\int_{L}^{\alpha_{-i}} (\xi - 2x_{-i} + \nu_i - 2\nu_{-i}) f(\xi) d\xi. \tag{68}$$

At

$$\xi = \alpha_{-i} = 3x_{-i} + 2\nu_{-i} - \nu_i. \tag{69}$$

We have

$$\xi - 2x_{-i} - 2\nu_{-i} + \nu_i = x_i > 0. \tag{70}$$

For small ξ the term in the integral can be negative. Thus, we cannot determien the sign of $\frac{\partial p_{-i}}{\partial x_{-i}}$ in general. Nevertheless, we can determine the response of x_i to a change in x_{-i}.

Taking the derivative of (64) with respect to x_i, we get

$$\frac{\partial x_i}{\partial x_{-i}} = -\frac{1}{2}. \tag{71}$$

Repeating the process by taking the derivative (65) with respect to x_i yields

$$\frac{\partial x_{-i}}{\partial x_i} = -\frac{1}{2}. \tag{72}$$

Thus, with the inclusion of forward markets, if (68) is positive, total capacity increases, and if (68) is negative, total capacity decreases. As we see in the numerical experiments, we can generate cases that lead to (68) having either sign.

5.3.3 The interior solution on the forward market

The necessary equilibrium conditions on the forward market are given in appendix. They are amenable to a numerical treatment (see next section) but do not lead to any general property.

5.4 Conclusion

In contrast with the neat Allaz-Vila type result obtained in Section 3, the introduction of capacities destroys all properties of the forward market and hence does not permit us to arrive at general properties of the capacity market equilibrium. Neither the equilibrium on the forward market nor the capacity market is guaranteed to exist. The model is still amenable to numerical solution, which we do next.

6 Numerical investigation

In this section we illustrate the possible consequences of adding a forward market using numerical examples, that is, moving from the two to three-stage model. Specifically, we show that a forward market can increase or decrease investments, with the result that its final impact on market power is ambiguous. The section is organized as follows. We first introduce a test problem based on realistic situations found in Europe. We then consider the case of an asymmetric situation that can relate to the competition between coal and gas utilities. We then take up two symmetric problems that we can relate to the competition between two utilities operating the same types of plants.

6.1 The test data

6.1.1 Demand assumptions

Consider a reference system with annual average hourly demand of 60 GW. We introduce a randomized demand function as follows. Suppose an instantanenous (in fact hourly) demand function

$$p = \xi' - \beta q'$$

where ξ' is uniformly distributed in an interval $[L, U]$

p is expressed in €/Mwh

q' is expressed in Gwh

In order to calibrate the system we assume that hourly demand varies randomly (or depending on the time of the year) between 40 and 80 Gwh at a price of 50 €/Mwh as a result of ξ taking its value in $[L, U]$. This is stated as

$$50 = \xi' - \beta q', \quad q' \in [40, 80].$$

Let $q'(\xi)$ be the value of q' when the price is 50 €/Mwh. Assuming an elasticity of .2 at the point $p = 50$ €/Mwh, $q = 50Gwh$ and a constant β, we impose

$$.2 = \frac{1}{\beta}$$

or $\beta = 5$, and obtain

$$\begin{aligned} \xi_L &= 50 + 40 \times 5 = 250 \\ \xi_U &= 50 + 80 \times 5 = 450. \end{aligned}$$

One can easily check that this corresponds to an elasticity decreasing from .25 to .125 when ξ increases from ξ_L to ξ_U, the corresponding demand is 40 and 80 Gwh and the price remains 50 €/Mwh, a behavior that is realistic. We can then rewrite the system

$$p = \xi - 5q'$$

as

$$p = \xi - q$$

by measuring q in 200 Mwh: a demand of 40 Gwh corresponds to 200 "200 Mwh". With $\xi = \xi_L = 250$ this gives a price of 250-200 = 50 €/Mwh.

6.1.2 Cost assumptions

We consider a market with two technologies, namely coal and combined-cycle gas turbines. The cost assumptions used for these technologies are taken from IEA (2005, table A10.2 page 227) after rounding. The annual fixed costs of the CCGT and Coal plants are obtained as follows

CCGT 5.75 (“Cost of Capital”) + 2.33 (“Fixed O and M Costs”) $\sim$ 8 €/Mwh

Coal 12.65 (“Cost of Capital”) + 3.50 (“Fixed O and M Costs”) $\sim$ 16 €/Mwh

Fuels costs are then established as

CCGT 19.6 (“Fuel Costs”) + 1.5 (“Variable O and M cost”) + 7.344 (“CO_2 cost”) $\sim$ 28 €/Mwh

Coal 14.93 (“Fuel Cost”) + 3.3 (“Variable O and M cost”) + 17.028 (“CO_2 cost”) $\sim$ 35 €/Mwh

These figures are based on gas and coal prices of 3 and 1.66 €/GJ respectively (which correspond roughly to 3 and 1.66 \$/MMbtu). These data can easily be updated to reflect current conditions. We leave a systematic analysis of competitive conditions to a further paper and retain the IEA assumptions in this work.

6.2 Solution approach

The model takes the form of nonlinear equations that are solved in EXCEL. The nonlinear equations are based on expressions that assume $\alpha_{-i} < \alpha_i$. It is not known in advance which plant reaches its capacity limit first in operations and hence whether i is associated with coal or gas units. We thus proceed by assuming an assignment of coal and gas to i and $-i$ respectively (intuitively coal should reach its capacity limit before gas) and verify afterwards that the inequality $\alpha_{-i} < \alpha_i$ is satisfied. Note that we can think of three sets of necessary conditions that correspond to

$$\begin{aligned} \alpha_{\text{gas}} &< \alpha_{\text{coal}} \\ \alpha_{\text{coal}} &< \alpha_{\text{gas}} \\ \alpha_{\text{coal}} &= \alpha_{\text{gas}} \end{aligned}$$

6.3 Asymmetric costs

We solved the necessary equilibrium conditions for both the capacity expansion model without and with a forward market and tested that we found a true equilibrium through varying the solutions and using second-order conditions in the futures market. Results are given after rescaling to the original units (that is, in GW of installed capacities and Gwh of hourly production).

	Capacity (in Gw)	α	Profit (10^6 €/h)
Gas	25.43	393.4	2.390
Coal	22.22	375.3	1.848
Total	47.65		4.238

Table 7.1.: Equilibrium without futures market

In this solution the player with the gas capacity builds more than the coal player, has 30 % higher profits, and operates below capacity for higher values of ξ than the coal player.

	Capacity (in Gw)	Futures in Gwh	α	Profit (10^6 €/h)
Gas	24.11	0	393.5	1.911
Coal	24.88	16.6	250	1.985
Total	49			3.896

Table 7.2.: First equilibrium with a futures market

The introduction of a futures market slightly increases the invested capacity. The level at which gas capacity is fully utilized is $\xi \geq 393.5$ while the coal capacity is fully utilized for all levels of ξ. Total profits drop to 3.896 10^6 €/hour. However, the coal player increases its profits at the expense of the gas player.

Profits are huge (profits of 4 10^6 €/h for an hourly demand of 50 Gwh lead to profits of 80 €/Mwh. This is due to the low (in absolute value) elasticity (from .25 to .125) for a long term problem and the Cournot assumption. Even though these values seen unrealistic for a long term problem, they correspond to those obtained by most authors when looking at market power.

The equilibrium with a futures market is a corner solution in that the coal player takes a futures position that fully utilizes all of its capacity for all potential demand levels and drives the other player from the futures market. The coal player comes out ahead of the gas player and garners greater profits than in a situation with no futures market. This solution is anomalous in that the higher-cost player increases its position at the expanse of the lower-cost player. This can happen because a large futures position can completely block the other player from entering the futures market. That is, the Cournot assumption that the other player does not respond in the futures game actually obtains in this case.

It is also true that the other corner solution is an equilibrium with gas capacity operating for all levels of ξ and the coal player out of the futures market. This equilibrium is shown in the following table. ù

	Capacity (in Gw)	Futures in Gwh	α	Profit (10^6 €/h)
Gas	27.43	40	250	2.629
Coal	19.82	0	370.4	1.389
Total	47.25			4.019

Table 7.3.: Second equilibrium with a futures market

Note that in this corner solution total capacity declines from the case with no futures equilibrium. Thus, with the parameters we have chosen, we get two corner equilibria, one where total capacity is increased and the other where total capacity is decreased. We see that anything can happen to total capacity within the same example.

One of the issues raised with the Allaz Vila model without capacity limits is that the decision to enter or not enter the futures market is a Prisoners dilemma game because both players are worse off. However, with the corner equilibrium, the result is not a Prisoners dilemma solution because the player that operates at capacity for all alphas improves its profit at the expense of the other player.

The following table contains the prices at the upper limit, U, and the lower limit, L, on the probability distribution.

	U	L
No futures equilibrium	212	104
First futures equilibrium	205	77
Second futures equilibrium	214	74

Table 7.4.: Priceis for the upper and lower limits of the probability distribution

We see that addings a forward market can either raise or lower the price at the upper limit of the probability distribution, depending on the change in total capacity. In both cases the price is lower at the lower levels of demand because of positive futures position increases spot production when capacity is not binding (the usual Allaz-Vila phenomenon). The price at the upper limit gives a sense of the effect of adding a forward market during the peak period in electricity generation because capacity is at or near capacity in the peak period. The effect on prices in base-load periods of a load duration curve would not be as dramatic as our results because there is a separate futures market for the base-load time slices and a corner solution is unlikely to occur then.

6.4 Symmetric costs

We consider two symmetric cases where both players have the same costs. We represent respectively the competition of two coal and two gas firms.

6.4.1 Competition of two gas firms ($k = 8, \nu = 28$)

We solve for the capacity equilibrium both without and with forward markets. With these parameters we find two equilibria. However, one is interior and one is at a corner. The interior solution is almost a corner solution. We checked the validity of the interior solution by varying the y's and the x's and found that the profit is at its peak in both the futures and capacity games. The result holds even though the profit difference is in the eighth decimal place between the interior and the corner solutions. The second-order conditions for the futures market for each player also hold. The results are as follows

	Capacity (in Gwh)	α	Profits in 10^6 €
Gas 1	24.36	393.4	2.241
Gas 2	24.36	393.4	2.241
Total	48.72		4.482

Table 7.5: Equilibrium without forward market

The solution without a futures market is symmetric. However, adding a futures market leads to an asymmetric equilibrium. Since either player can be labeled Gas 1, an asymmetric equilibrium implies two possible equilibria.

	Capacity (in Gwh)	Futures in Gwh	α	Profits in 10^6 €
Gas 1	25.44	.002	393.42	2.174
Gas 2	22.20	11.1	250.04	2.253
Total	47.64			4.427

Table 7.6: First equilibrium with a forward market

Here the introduction of the forward market decreases the total investment in this equilibrium.

	Capacity (in Gwh)	Futures in Gwh	α	Profits in 10^6 €
Gas 1	22.78	0	393.42	1.669
Gas 2	27.52	30	250	2.427
Total	50.30			4.096

Table 7.7: Second equilibrium with a forward market

Again, we checked to make sure this is an equilibrium by varying the x's around the solution. Unlike the interior solution, total capacity increases. The next table presents the prices.

	U	L
No futures equilibrium	206	102
First futures equilibrium	212	84
Second futures equilibrium	198	70

Table 7.8: Prices for the upper and lower limits of the probability distribution

As before the corner solution can lead to higher or lower prices at U and leads to lower prices at L.

6.4.2 Competition between coal firms ($k = 16, \nu = 35$)

As in the other examples, we solve for the necessary equilibrium conditions for the capacity expansion model both without and with a forward market and check that we have a solution by varying values around the optimum and usign the second-order conditions for interior solutions. The results are given in Tables 7.9 and 7.10.

	Capacity (in Gwh)	α	Profits in 10^6 €
Coal 1	22.33	379	2.022
Coal 2	22.33	379	2.022
Total	44.66		4.044

Table 7.9: Equilibrium without forward market

	Capacity (in Gwh)	Futures in Gwh	α	Profits in 10^6 €
Coal 1	21.05	0	370.1	1.600
Coal 2	24.91	15.87	250	2.172
Total	45.96	3.772		

Table 7.10: Equilibrium with forward market

In this case, a futures market increases capacity.

	U	L
No futures equilibrium	227	107
With futures equilibrium	220	93

Table 7.11: Prices for the upper and lower limits of the probability distribution

6.5 Another symmetric case

So far, we have not presented a case with just an interior solution and no boundary solution. We now present such a case. Here we use the costs for the coal pant and reduce L to 50 from 250. The effect of lowering L increases the cost of being at capacity for each ξ because prices are very low at the low ξ and production is much higher than would be the case at the duopoly solution for that ξ.

Solving this case for the market without and with a forward market, we obtain

	Capacity (in Gwh)	α	Profits in 10^6 €
Caal 1	20.12	336.86	1.082
Coal 2	20.12	336.86	1.082
Total	40.24		2.164

Table 7.12: Equilibrium without forward market

	Capacity (in Gwh)	Futures in Gwh	α	Profits in 10^6 €
Coal 1	21.22	3.81	315.92	1.103
Coal 2	17.60	5.19	266.46	.981
Total	38.82			2.084

Table 7.13: Equilibrium with forward market

To check that there is no corner equilibrium, we did the following. In our model we set $y_i = 0$ and y_{-i} to a large number so that we have the corner solution for a range of capacity levels. We then varied the x's to find the capacity equilibrium given the corner solution from the futures position. We then tested to see if these capacities could produce a corner equilibrium in the futures market. We found that at these capacities player $-i$, in optimizing its futures position, reduced y_{-i} below the level necessary to have a corner solution. Thus, there is no corner equilibrium with these parameters.

In this case, the futures market leads to a decrease in total capacity.

7 Conclusions

Market power is a recurrent concern in restructured electricity markets. The common wisdom is that incumbent generation companies have market power and will eventually exercise it. Resource adequacy is an emerging concern: restructured electricity markets may not provide sufficient incentives for investments. Market power may add to the effect, as restricting capacities is an obvious way to exercise and reinforce market power. Forward contracts have appeared as an ingenious remedy in that context. Besides offering hedging possibilities, they are commonly seen as good instruments to mitigate market power, Joskow (2006) and Wolak (2000). Following the seminal contribution of Allaz and Allaz-Vila, a whole stream of literature argues that position. We show that the situation is much less clear than usually assumed.

The good properties of long-term contracts have indeed been established under ideal situations; they are either eoxgenously given as in the early electricity literature, or endogenously determined in a market with infinite capacities. We show that endogenously limiting capacities can destroy the ability of forward contracts to mitigate market power. In Part 1 we indicated that forward contracts have no effect when future demand is known. We prove here that they have an undetermined effect when demand is unknown at the time the investment and forward positions are taken.

Although we do not have a load curve in our model, the results are broadly applicable to pricing at the peak, the time of day when markets are most susceptible to market power. Given the high levels of demand at or near the peak, corner solutions can create opportunities for a player to keep other players out of the futures market and potentially limit capacity to levels below what would be case without a futures amrket.

Our results also show the conceptual difficulties of making broad conclusions about complicated markets using simple models. We have results that lead to the three possible outcomes that can occur by making natural modifications to models. Allaz and Vila show that futures markets increase competition. Adilov (2005) shows that additing a capacity constraint increases market power using a binomial distribution of demand. We show that the result is ambiguous when we use the assumption of a continuous demand distribution. This serves as a caution when generalizing theoretical results in modeling abstractions as the basis for forming government policy.

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Appendix

The appendix consists of two parts. Appendix A1 reports the formula of the interior equilibrium solution. Appendix A2 specializes all equilibrium formula to the case of the uniform distribution.

Appendix A1

Computation of $\frac{\partial p_i}{\partial x_i}$ and $\frac{\partial p_{-i}}{\partial x_{-i}}$

$$\frac{\partial p_i}{\partial x_i} = \int_{\alpha_i}^{\infty} (\xi - 2x_i - x_{-i} - \nu_i) f(\xi) d\xi - k_i \tag{A.1}$$

and

$$\begin{aligned}\frac{\partial p_{-i}}{\partial x_{-i}} &= \frac{1}{2}\int_{\alpha_{-i}}^{\alpha_i} (\xi - 2x_{-i} + \nu_i - 2\nu_{-i} - y_i) f(\xi) d\xi \\ &+ \int_{\alpha_i}^{\infty} (\xi - x_i - 2x_{-i} - \nu_{-i}) f(\xi) d\xi - k_{-i}\end{aligned} \tag{A.2}$$

Computation of $\frac{\partial p_i}{\partial y_{-i}}$ and $\frac{\partial p_{-i}}{\partial y_i}$

We have respectively

$$\begin{aligned}\frac{\partial p_i}{\partial y_{-i}} &= -\frac{1}{9}\int_0^{\alpha_{-i}} (2\xi + y_i - 2y_{-i} - 4\nu_i + 2\nu_{-i}) f(\xi) d\xi & \text{(A.3)} \\ \frac{\partial p_{-i}}{\partial y_i} &= \frac{1}{9}\int_0^{\alpha_{-i}} (\xi - y_i - 4y_{-i} + \nu_i - 2\nu_{-i}) f(\xi) d\xi) & \text{(A.4)} \\ &- \frac{x_{-i}}{2}\int_{\alpha_{-i}}^{\alpha_i} f(\xi) d\xi.\end{aligned}$$

Computation of $\frac{\partial y_i}{\partial x_{-i}}$ and $\frac{\partial y_{-i}}{\partial x_i}$

These expressions are obtained by perturbing x_i and x_{-i} in the forward market equilibrium conditions

$$\frac{\partial p_i}{\partial y_i} = \frac{\partial p_{-i}}{\partial y_{-i}} = 0.$$

This is done as follows

(i) Perturbations with respect to x_i

Consider first the equilibrium condition

$$\begin{aligned} 0 &= \frac{\partial p_i}{\partial y_i} \\ &= \frac{1}{9}\int_0^{\alpha_{-i}(x,y)} (\xi - 4y_i - y_{-i} - 2\nu_i + \nu_{-i}) f(\xi) d\xi - \frac{y_i}{2}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} f(\xi) d\xi. \end{aligned}$$

We write

$$\begin{aligned} 0 &= \frac{\partial^2 p_i}{\partial x_i \partial y_i} \\ &= \frac{1}{9}\int_0^{\alpha_{-i}(x,y)} - \left(4\frac{\partial y_i}{\partial x_i} + \frac{\partial y_{-i}}{\partial x_i}\right) f(\xi) d(\xi) \\ &+ \frac{1}{9}\left[(\alpha_{-i}(x,y) - 4y_i - y_{-i} - 2\nu_i + \nu_{-i}) f(\alpha_{-i}(x,y))\right]\frac{\partial \alpha_{-i}}{\partial x_i} \\ &- \frac{1}{2}\frac{\partial y_i}{\partial x_i}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} f(\xi) d\xi - \frac{y_i}{2}\left[f(\alpha_i)\frac{\partial \alpha_i}{\partial x_i} - f(\alpha_{-i})\frac{\partial \alpha_{-i}}{\partial x_i}\right] \end{aligned}$$

or

$$\begin{aligned} &\left(4\frac{\partial y_i}{\partial x_i} + \frac{\partial y_{-i}}{\partial x_i}\right)\int_0^{\alpha_{-i}(x,y)} f(\xi) d\xi \\ &+ \frac{9}{2}\frac{\partial y_i}{\partial x_i}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} f(\xi) d\xi - 9y_i f(\alpha_i) = 0 \end{aligned} \tag{A.5}$$

This gives a first relation involving $\frac{\partial y_i}{\partial x_i}$ and $\frac{\partial y_{-i}}{\partial x_i}$.

Consider now the equilibrium condition

$$\begin{aligned} 0 &= \frac{\partial p_{-i}}{\partial y_{-i}} \\ &= \int_0^{\alpha_{-i}(x,y)} (\xi - y_i - 4y_{-i} + \nu_i - 2\nu_{-i}) f(\xi) d\xi = 0. \end{aligned}$$

We write

$$\begin{aligned} 0 &= \frac{\partial^2 p_{-i}}{\partial x_i \partial y_{-i}} \\ &= -\left(\frac{\partial y_i}{\partial x_i} + 4\frac{\partial y_{-i}}{\partial x_i}\right)\int_0^{\alpha_{-i}(x,y)} f(\xi) d\xi = 0 \end{aligned}$$

or

$$\frac{\partial y_i}{\partial x_i} + 4\frac{\partial y_{-i}}{\partial x_i} = 0 \tag{A.6}$$

which is a second relation involving $\frac{\partial y_i}{\partial x_i}$ and $\frac{\partial y_{-i}}{\partial x_i}$.

For the particular case of the uniform distribution the relations reduce to

$$\left(4\frac{\partial y_i}{\partial x_i}+\frac{\partial y_{-i}}{\partial x_i}\right)(\alpha_{-i}-L)-\frac{9}{2}\frac{\partial y_i}{\partial x_i}(\alpha_i-\alpha_{-i})+9y_i=0$$

and

$$\frac{\partial y_i}{\partial x_i}+4\frac{\partial y_{-i}}{\partial y_i}=0.$$

(ii) Perturbation with respect to x_{-i}

Consider again the equilibrium condition $\frac{\partial p_i}{\partial y_i}=0$. We write

$$\begin{aligned}
0 &= \frac{\partial^2 p_i}{\partial x_{-i}\partial y_i}\\
&= -\frac{1}{9}\left(4\frac{\partial y_i}{\partial x_{-i}}+\frac{\partial y_{-i}}{\partial x_{-i}}\right)\int_0^{\alpha_{-i}(x,y)} f(\xi)d\xi\\
&+ \frac{1}{9}(\alpha_{-i}(x,y)-4y_i-y_{-i}-2\nu_i+\nu_{-i})f\left(\alpha_{-i}(x,y)\right)\frac{\partial\alpha_{-i}}{\partial x_{-i}}\\
&- \frac{1}{2}\frac{\partial y_i}{\partial x_{-i}}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} f(\xi)d\xi-\frac{y_i}{2}\left[f(\alpha_i(x,y)\frac{\partial\alpha_i}{\partial x_{-i}}-f\left(\alpha_{-i}(x,y)\frac{\partial\alpha_{-i}}{\partial x_{-i}}\right)\right]
\end{aligned}$$

or

$$\begin{aligned}
&\left(4\frac{\partial y_i}{\partial x_{-i}}+\frac{\partial y_{-i}}{\partial x_{-i}}\right)\int_0^{\alpha_{-i}(x,y)} f(\xi)d\xi\\
&- 3(\alpha_{-i}-4y_i-y_{-i}-2\nu_i+\nu_{-i})f(\alpha_{-i})\\
&+ \frac{9}{2}\frac{\partial y_i}{\partial x_{-i}}\int_{\alpha_{-i}}^{\alpha_i} f(\xi)d\xi\\
&- \frac{27}{2}y_i[f(\alpha_{-i})]=0\\
&+ \frac{9}{2}y_i[f(\alpha_i)]=0
\end{aligned}\tag{A.7}$$

which is a first relation involving $\frac{\partial y_i}{\partial x_{-i}}$ and $\frac{\partial y_{-i}}{\partial x_{-i}}$.

Turning now to the equilibrium condition

$$\begin{aligned}
0 &= \frac{\partial p_{-i}}{\partial y_{-i}}\\
&= \int_0^{\alpha_{-i}(x,y)}(\xi-y_i-4y_{-i}+\nu_i-2\nu_{-i})f(\xi)d\xi.
\end{aligned}$$

We write

$$\begin{aligned} 0 &= \frac{\partial^2 p_{-i}}{\partial x_{-i}\partial y_{-i}} \\ &= \int_0^{\alpha_{-i}(x,y)} - \left(\frac{\partial y_i}{\partial x_{-i}} + 4\frac{\partial y_{-i}}{\partial x_{-i}}\right) f(\xi)d\xi \\ &+ (\alpha_{-i}(x,y) - y_i - 4y_{-i} + \nu_i - 2\nu_{-i}) f(\alpha_{-i}) \frac{\partial \alpha_{-i}}{\partial x_{-i}} \\ &- \left(\frac{\partial y_i}{\partial x_{-i}} + 4\frac{\partial y_{-i}}{\partial x_{-i}}\right) \end{aligned} \tag{A.8}$$

or

$$\int_0^{\alpha_{-i}(x,y)} f(\xi)d\xi + 3(\alpha_{-i}(x,y) - y_i - 4y_{-i} + \nu_i - 2\nu_{-i}) f(\alpha_{-i}) = 0 \tag{A.9}$$

which is a second relation involving $\frac{\partial y_i}{\partial x_{-i}}$ and $\frac{\partial y_{-i}}{\partial x_{-i}}$.

Appendix 2: The uniform distribution

The appendix reports all formula relative to the treatment of the uniform distribution.

Appendix A2.1

This gives for the particular case of uniform distribution

$$\begin{aligned} \frac{\partial p_i}{\partial y_{-i}} &= -\frac{1}{9(U-L)}\left\{2\left[\frac{\xi^2}{2}\right]_L^{\alpha_{-i}} + (y_i - 2y_{-i} - 4\nu_i + 2\nu_{-i})(\alpha_{-i} - L)\right\} \\ &= -\frac{1}{9(U-L)}\left[(\alpha_{-i}^2) - L^2) + (y_i - 2y_{-i} - 4\nu_i + 2\nu_{-i})(\alpha_{-i} - L)\right] \end{aligned}$$

$$\begin{aligned} \frac{\partial p_{-i}}{\partial y_i} &= \frac{1}{9(U-L)}\left\{\left(\frac{\xi^2}{2}\right)_L^{\alpha_{-i}} - (y_i + 4y_{-i} - \nu_i + 2\nu_{-i})(\alpha_{-i} - L)\right\} \\ &- \frac{x_{-i}}{2(U-L)}(\alpha_i - \alpha_{-i}) \\ \\ &= \frac{1}{(U-L)}\left\{\frac{1}{18}(\alpha_{-i}^2 - L^2) - 19(y_i + 4y_{-i} - \nu_i + 2\nu_{-i})(\alpha_{-i} - L)\right\} \\ &- \frac{x_{-i}}{2(U-L)}(\alpha_i - \alpha_{-i}). \end{aligned}$$

For the particular case of the uniform distribution, these relations reduce to

$$\begin{aligned} & \left(4\frac{\partial y_i}{\partial x_{-i}} + \frac{\partial y_{-i}}{\partial x_{-i}}\right)(\alpha_{-i} - L) - 3(\alpha_{-i} - 4y_i - y_{-i} - 2\nu_i + \nu_{-i}) \\ + & \frac{9}{2}\frac{\partial y_i}{\partial x_{-i}}(\alpha_i - \alpha_{-i}) - \frac{27}{2}y_i = 0 \end{aligned}$$

and

$$\left(\frac{\partial y_i}{\partial x_{-i}} + 4\frac{\partial y_{-i}}{\partial x_{-i}}\right)(\alpha_{-i} - L) + 3(\alpha_{-i} - y_i - 4y_{-i} + \nu_i - 2\nu_{-i}) = 0.$$

Appendix A2.2: The pure capacity market: first and second order conditions

Take $f(\xi) = \frac{1}{U - L}$ where L and U indicate lower and upper bound of ξ. We have for $\frac{\partial p_i}{\partial x_i}$

$$\begin{aligned} & \int_{\alpha_i}^{\infty} (\xi - 2x_i - x_{-i} - \nu_i) f(\xi) d\xi - k_i \\ = & \frac{1}{U - L}\left[\frac{\xi^2}{2}\right]_{\alpha_i}^{U} - \frac{U - \alpha_i}{U - L}(2x_i + x_{-i} + \nu_i) - k_i = 0 \end{aligned}$$

or

$$\frac{1}{2(U - L)}(U^2 - \alpha_i^2) - \frac{U - \alpha_i}{U - L}(2x_i + x_{-i} + \nu_i) - k_i = 0.$$

or

$$\frac{U^2 - \alpha_i^2}{2} - (U - \alpha_i)(2x_i + x_{-i} + \nu_i) - k_i(U - L) = 0.$$

Note that $\alpha_i = 2x_i + x_{-i} + \nu_i$ and hence

$$\frac{U^2 - \alpha_i^2}{2} - (U - \alpha_i)\alpha_i - k_i(U - L) = 0$$

or

$$(U - \alpha_i)\left[\frac{U + \alpha_i}{2} - \alpha_i\right] - k_i(U - L) = 0$$

or

$$(U - \alpha_i)^2 = 2k_i(U - L) \Rightarrow \alpha_i = U \pm \sqrt{2k_i(U - L)}.$$

The second order condition for the equilibrium is

$$\frac{\partial}{\partial x_i}\left[U - \alpha_i)^2 - 2k_i(U - L)\right] \leq 0$$

or

$$2(U - \alpha_i)(-2) \leq 0$$

or

$$U - \alpha_i \geq 0.$$

Therefore the equilibrium is reached for $\overline{\alpha}_i = U - \sqrt{2k_i(U-L)}$.
We also have for $\dfrac{\partial p_{-i}}{\partial x_{-i}}$

$$\begin{aligned}
& \frac{1}{2}\int_{\alpha_{-i}}^{\alpha_i} (\xi - 2x_{-i} + \nu_i - 2\nu_{-i}) f(\xi) d\xi \\
= \; & \frac{1}{2(U-L)}\left[\frac{\alpha_i^2 - \alpha_{-i}^2}{2} - (2x_{-i} - \nu_i + 2\nu_{-i})(\alpha_i - \alpha_{-i})\right]
\end{aligned}$$

$$\begin{aligned}
& \int_{\alpha_i}^{\infty} (\xi - x_i - 2x_{-i} - \nu_{-i}) f(\xi) d\xi \\
= \; & \frac{1}{U-L}\left[\frac{U^2 - \alpha_i^2}{2} - (x_i + 2x_{-i} + \nu_i)(U - \alpha_i)\right].
\end{aligned}$$

In total

$$\begin{aligned}
& \frac{1}{U-L}\Bigg[\frac{1}{4}(\alpha_i^2 - \alpha_{-i}^2) - \frac{\alpha_i - \alpha_{-i}}{2}(2x_{-i} - \nu_i + 2\nu_{-i}) \\
+ \; & \frac{U^2 - \alpha_i^2}{2} - (x_i + 2x_{-i} + \nu_{-i})(U - \alpha_i) - k_{-i}\Bigg] = 0.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial p_{-i}}{\partial x_{-i}} = \; & \frac{1}{2}\left[\left(\frac{\alpha_i^2 - \alpha_{-i}^2}{2}\right) - (\alpha_i - \alpha_{-i})(2x_{-i} - \nu_i + 2\nu_{-i})\right] \\
+ \; & \frac{U^2 - \alpha_i^2}{2} - (U - \alpha_i)(x_i + 2x_i + \nu_{-i}) - k_{-i}(U - L).
\end{aligned}$$

The first order condition can thus be restated as

$$\begin{aligned}
& [3(\alpha_i - U)^2 + 3U^2 - 2\alpha_i(2\nu_{-i} - \nu_i) - 12k_{-i}] \\
- \; & 2[3U - \alpha_i - (2\nu_{-i} - \nu_i)]\alpha_{-i} + \alpha_{-i}^2 = 0.
\end{aligned}$$

In order to verify the second order condition, we derive the expression. This gives with respect to x_{-i}

$$\begin{aligned}
& 6(\alpha_i - U)\frac{\partial \alpha_i}{\partial x_{-i}} - 2(2\nu_{-i} - \nu_i)\frac{\partial \alpha_i}{\partial x_{-i}} \\
- \; & 2\alpha_{-i}\left(-\frac{\partial \alpha_i}{\partial x_{-i}}\right) - 2[3U - \alpha_i - (2\nu_{-i} - \nu_i)]\frac{\partial \alpha_{-i}}{\partial x_{-i}} + 2\alpha_{-i}\frac{\partial \alpha_{-i}}{\partial x_{-i}}
\end{aligned}$$

or

$$\begin{aligned} & 6(2x_i + x_{-i} + \nu_i - U) - 2(2\nu_{-i} - \nu_i) + 2(2x_{-i} + 2\nu_{-i} - \nu_i) \\ - & 6[3U - 2x_i - x_{-i} - \nu_i - 2\nu_{-i} + \nu_i) + 6(3x_{-i} + 2\nu_{-i} - \nu_i) \end{aligned}$$

Second order condition

$$\begin{aligned} \frac{\partial p_i}{\partial x_i} &= (U - \alpha_i)^2 - k_i \\ \frac{\partial p_{-i}}{\partial x_{-i}} &= [3(\alpha_i - U)^2 + 3U^2 - 2\alpha_i(2\nu_{-i} - \nu_i) - 12k_{-i}] \\ &- 2[3U - \alpha_i - (2\nu_{-i} - \nu_i)]\alpha_{-i} + \alpha_{-i}^2 \\ \frac{\partial^2 p_i}{\partial x_i^2} &= -2(U - \alpha_i)\left(\frac{\partial \alpha_i}{\partial x_i}\right) = -4(U - \alpha_i) \end{aligned}$$

always satisfied if

$$2x_i + x_{-i} + \nu_i \leq 0.$$

$$\frac{\partial^2 p_{-i}}{\partial x_{-i}^2} = \frac{\partial}{\partial \alpha_i}\left(\frac{\partial p_{-i}}{\partial x_{-i}}\right)\frac{\partial \alpha_i}{\partial x_{-i}} + \frac{\partial}{\partial \alpha_{-i}}\frac{\partial p_{-i}}{\partial x-i}\frac{\partial \alpha_{-i}}{\partial x_{-i}}$$

$$\frac{\partial_i}{\partial x_{-i}} = 1;\ \frac{\partial \alpha_{-i}}{\partial x_{-i}} = 3$$

$$\begin{aligned} & [3(\alpha_i - U)(2) - 2(2\nu_{-i} - \nu_i) + 2\alpha_{-i}] \\ - & 2[3U - \alpha_i - (2\nu_{-i} - \nu_i)]3 + 2\alpha_{-i}(3) \\ = & 6(2x_i + x_{-i} + \nu_i - U) - 2(2\nu_{-i} - \nu_i) + 6x_{-i} + 4\nu_{-i} - 2\nu_i \\ - & 6(3U - 2x_i - x_{-i} - \nu_i - 2\nu_{-i} + \nu_i) + 6(3x_{-i} + 2\nu_{-i} - \nu_i) \\ = & 12x_i + 6x_{-i} + 6\nu_i - 6U - 4\nu_{-i} + 2\nu_i + 6x_{-i} + 4\nu_{-i} - 2\nu_i \\ - & 18U + 12x_i + 6x_{-i} + 6\nu_i + 12\nu_{-i} - 6\nu_i + 18x_{-i} + 12\nu_{-i} - 6\nu_i \\ = & 24x_i + 36x_{-i} - 24U + 24\nu_{-i} \\ & 2x_i + 3x_{-i} - 2U + 2\nu_{-i} \end{aligned}$$

Second order verified if

$$x_i + \frac{3}{2}x_{-i} + \nu_{-i} \leq U$$

which results from

$$\alpha_{-i}(x) \leq \alpha_i(x) \leq U.$$

Appendix A2.3: the forward market

$$
\begin{aligned}
\frac{\partial p_i}{\partial y_i} &= \frac{1}{9}\left[\frac{(\alpha^2_{-i} - L^2}{2} - (\alpha_{-i} - L)(4y_i + y_{-i} - 2\nu_i + \nu_{-i}\right] - \frac{y_i}{2}(\alpha_i - \alpha_{-i}) = 0 \\
\frac{\partial p_i}{\partial y_{-i}} &= \frac{1}{9}\left[\frac{(\alpha^2_{-i} - L^2}{2} - (\alpha_{-i} - L)(y_i + 4y_{-i} + \nu_i - 2\nu_{-i})\right] = 0 \\
\alpha_{-i} &= 3x_{-i} + 2\nu_{-i} - \nu_i - (2y_{-i} - y_i) = a_{-i} - (2y_{-i} - y_i) \\
\alpha_i &= 2x_i + x_{-i} + \nu_i - y_i
\end{aligned}
$$

i) The corner solution

Note that

$$
\alpha_{-i} = L \quad of \quad \frac{\partial p_{-i}}{\partial y_{-i}} = 0
$$

always satisfy the first order conditions

ii) The non corner solution

Eliminating the corner solution, we rewrite the first order conditions as

$$
\begin{aligned}
&\frac{\alpha^2_{-i} - L^2}{2} - (\alpha_{-i} - L)(4y_i + y_{-i} - 2\nu_i + \nu_{-i}) - \frac{9}{2}y_i(\alpha_i - \alpha_{-i}) = 0 \\
&\frac{\alpha_{-i} + L}{2} - (y_i + 4y_{-i} + \nu_i - 2\nu_{-i}) = 0.
\end{aligned}
$$