# Stochastic Market Equilibrium Models Using Complementarity Theory *


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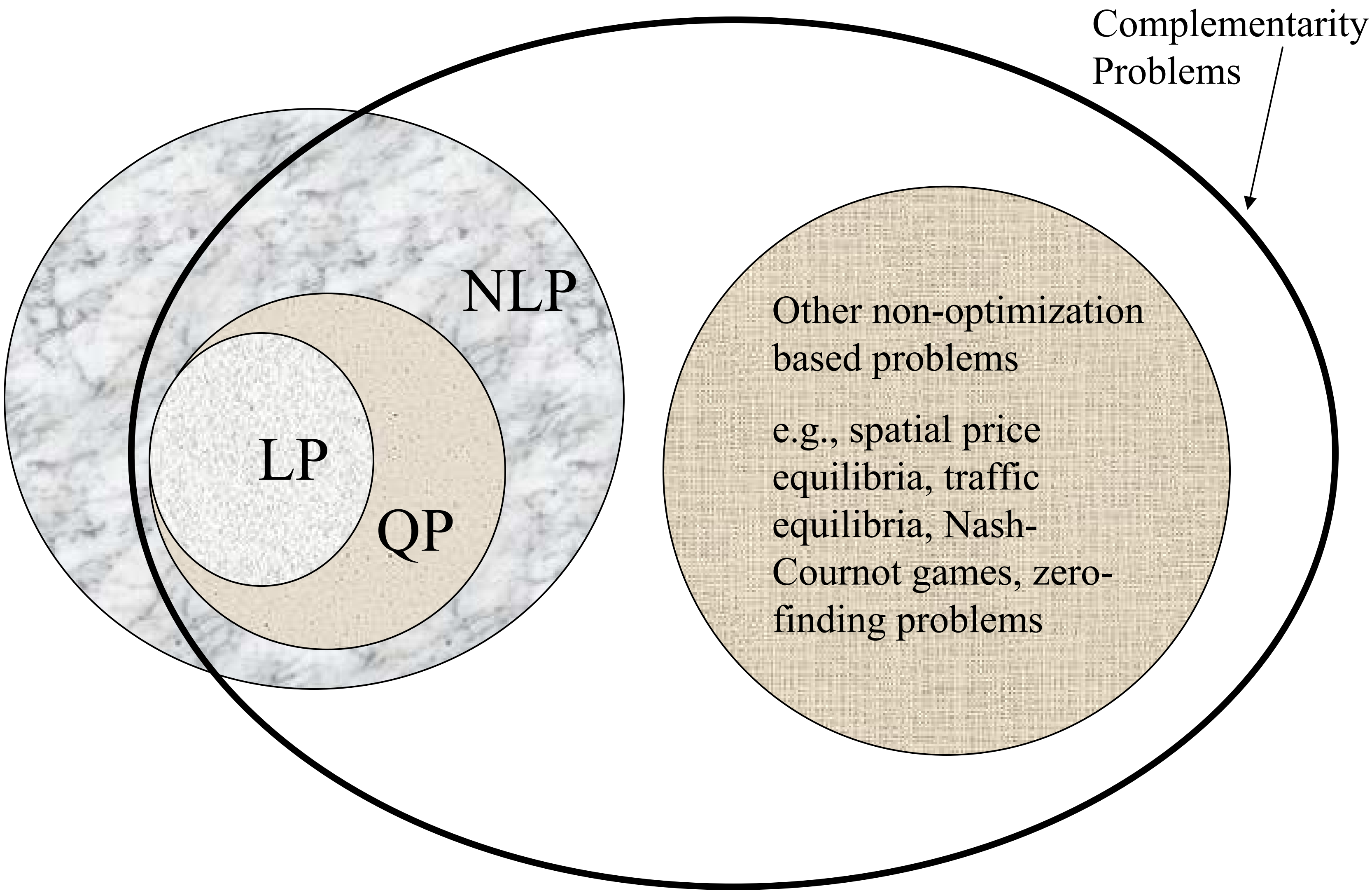
# Outline of Talk

- Complementarity problems
  - Overview
  - World Gas Model
  - Stochastic complementarity problem formulation for a small power market model
- Sketch of Benders algorithm (mention of Scenario Reduction Approach)
- Selected numerical results
- Ongoing Work
- References

# Complementarity Problems and Stochasticity

# Complementarity Problems vis-à-vis Optimization and Game Theory Problems



# Equilibrium Problems Expressed as Mixed Nonlinear Complementarity Problems

(Mixed) Nonlinear Complementarity Problem MNCP

Having a function $F: R^n \rightarrow R^n$, find an $x \in R^{n_1}$, $y \in R^{n_2}$ such that

$F_i(x,y) \geq 0, x_i \geq 0, F_i(x,y) * x_i = 0 \; for \; i = 1,\ldots,n_1$

$F_i(x,y) = 0, y_i$ free, $for \; i = n_1 + 1,\ldots,n$

Example

$$F(x_1,x_2,y_1) = \begin{pmatrix} F_1(x_1,x_2,y_1) \\ F_2(x_1,x_2,y_1) \\ F_3(x_1,x_2,y_1) \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - y_1 \\ x_1 + x_2 + y_1 - 2 \end{pmatrix}$$ so we want to find $x_1, x_2, y_1$ s.t.

$$x_1 + x_2 \geq 0 \qquad x_1 \geq 0 \qquad (x_1 + x_2) * x_1 = 0$$

$$x_1 - y_1 \geq 0 \qquad x_2 \geq 0 \qquad (x_1 - y_1) * x_2 = 0$$

$x_1 + x_2 + y_1 - 2 = 0 \quad y$ free

One solution: $(x_1, x_2, y_1) = (0, 2, 0)$, why? Any others?

# Nonlinear Programs Expressed as Mixed Nonlinear Complementarity Problems

Consider a generic nonlinear program and its resulting KKT conditions

$$\min\ f(x)$$

$$s.t.\ g_i(x) \le 0, i = 1,\ldots,m \quad (u_i)$$

$$h_j(x) = 0,\ j = 1,\ldots,p \quad (v_j)$$

KKT conditions, find $\bar{x} \in R^n, \bar{u} \in R^m, \bar{v} \in R^p$ $s.t.$

$$\left\{ \begin{array}{l} (i) \nabla f(\bar{x}) + \sum_{i=1}^{m} \bar{u}_i \nabla g_i(\bar{x}) + \sum_{j=1}^{p} \bar{v}_i \nabla h_j(\bar{x}) = 0 \\ (ii) g_i(\bar{x}) \le 0, \bar{u}_i \ge 0, g_i(\bar{x})\bar{u}_i = 0, \text{ for all } i = 1,\ldots,m \\ (iii) h_j(\bar{x}) = 0, \bar{v}_j \text{ free, for all } j = 1,\ldots,p \end{array} \right\}$$

# Nonlinear Programs Expressed as Mixed Nonlinear Complementarity Problems

Thus, we get a mixed NCP as follows:

$$F\begin{pmatrix} x \\ u \\ v \end{pmatrix} = \begin{pmatrix} \nabla f(x) + \sum_{i=1}^{m} u_i \nabla g_i(x) + \sum_{j=1}^{p} v_j \nabla h_j(x) \\ -g_i(x), i = 1, \ldots, m \\ h_j(x), \, j = 1, \ldots, p \end{pmatrix}$$

$$\nabla f(x) + \sum_{i=1}^{m} u_i \nabla g_i(x) + \sum_{j=1}^{p} v_j \nabla h_j(x) = 0 \qquad x \text{ free}$$

$$-g_i(x) \geq 0, i = 1, \ldots, m \qquad u_i \geq 0, \left(-g_i(x)\right) * u_i = 0$$

$$h_j(x) = 0, \, j = 1, \ldots, p \qquad v_j \text{ free}$$

## Producer Duopoly Expressed as Nonlinear Complementarity Problems

-Two producers competing with each other on how much to produce given as $q_i, i = 1, 2$

- Market Inverse demand function

$$p(q_1 + q_2) = \alpha - \beta(q_1 + q_2), \text{ where } \alpha, \beta > 0$$

that the producers can manipulate by their production

- Production cost function

$$c_i(q_i) = \gamma_i q_i, i = 1, 2, \text{ where } \gamma_i > 0$$

# Producer Duopoly Expressed as Nonlinear Complementarity Problems

Producer 1's optimization problem:

$$\max \ \left(\alpha - \beta(q_1 + q_2)\right) * q_1 - \gamma_1 q_1$$

$$s.t. \ q_1 \geq 0$$

KKT conditions:

Find $q_1$ s.t. $2\beta q_1 + \beta q_2 - \alpha + \gamma_1 \geq 0 \quad q_1 \geq 0 \quad \left(2\beta q_1 + \beta q_2 - \alpha + \gamma_1\right) q_1 = 0$

For Producer 2, similar idea, that is:

Find $q_2$ s.t. $2\beta q_2 + \beta q_1 - \alpha + \gamma_2 \geq 0 \quad q_1 \geq 0 \quad \left(2\beta q_2 + \beta q_1 - \alpha + \gamma_2\right) q_1 = 0$

Need to solve both at same time (why?) to get the resulting pure NCP

$$F\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 2\beta q_1 + \beta q_2 - \alpha + \gamma_1 \\ 2\beta q_2 + \beta q_1 - \alpha + \gamma_2 \end{pmatrix}$$

Can generalize to $N$ players, will get a Nash-Cournot equilibrium

# Example of an Equilibrium Problem
# Energy Market Equilibria: PIES
# (Cottle, Pang, Stone)

- As a result of the energy crisis in the US in the mid 1970's the Project Independence Evaluation System (PIES) energy model was developed
- Models a competitive market with two sets of players (agents): suppliers and consumers
- Given a perceived demand, suppliers solve a related LP
- Consumers demand is a function of all energy prices and given by an econometrically-derived demand equation
- Several later versions: Intermediate Future Forecasting System (1980's), National Energy Modeling System (1990's-present)

# Example of an Equilibrium Problme
# Energy Market Equilibria: PIES

1. Supply Side

$\min\ c^T x$ ! total cost of production

$s.t.$

$Ax \geq q$ ! demand, dual price: $\pi$

$Bx \geq b$ !non-demand

$x \geq 0$

where

$c =$ vector of prod. costs

$q =$ demand quantities

# Example of an Equilibrium Problem
# Energy Market Equilibria: PIES

2. Demand Side

$$\ln\left(\frac{q_i}{q_i^0}\right)=\sum_{j=1}^{n} e_{ij}\ln\left(\frac{p_i}{p_i^0}\right) \text{ or}$$

$$q_i(p)=q_i^0\prod_{j=1}^{n}\left(\frac{p_i}{p_i^0}\right)^{e_{ij}}$$

where

$q_i^0$ = reference demand for product i

$p_i^0$ = reference price for product i

$e_{ij}$ = elasticities

(3) Equilibrating condition

$$\pi^* = p^*$$

# Equilibrium Problems Expressed as Mixed Nonlinear Complementarity Problems

PIES is an example of a pure NCP

Conditions taken component-wise or by vectors it's the same, why?

$$c - A^T\pi - B^T\gamma \geq 0 \quad x \geq 0 \quad \left(c - A^T\pi - B^T\gamma\right)^T x = 0$$

$$Ax - q(\pi) \geq 0 \quad \pi \geq 0 \quad \left(Ax - q(\pi)\right)^T \pi = 0$$

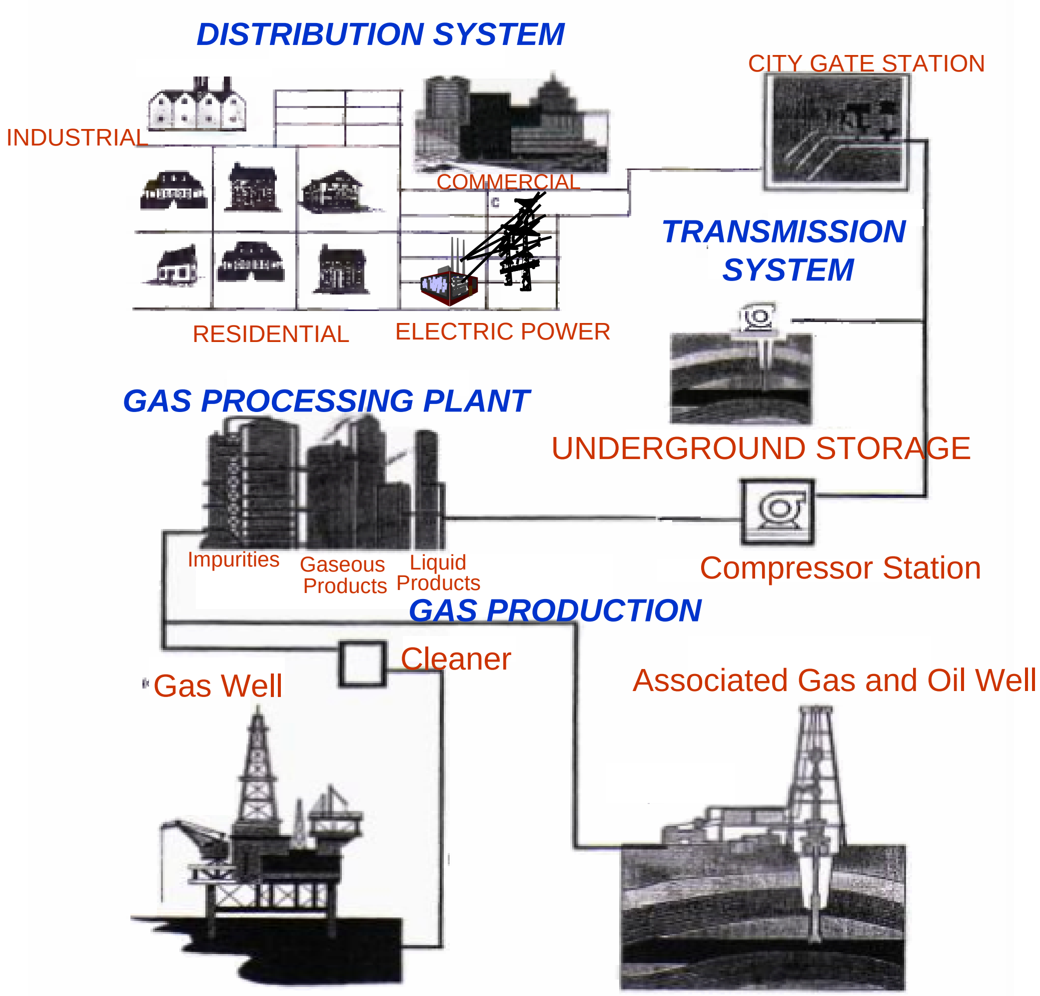
$$Bx - b \geq 0 \quad \gamma \geq 0 \quad \left(Bx - b\right)^T \gamma = 0$$

Thus, the function $F$ is defined as follows:

$$F\begin{pmatrix} x \\ \pi \\ \gamma \end{pmatrix} = \begin{pmatrix} c - A^T\pi - B^T\gamma \\ Ax - q(\pi) \\ Bx - b \end{pmatrix}$$

# World Gas Model- Overview

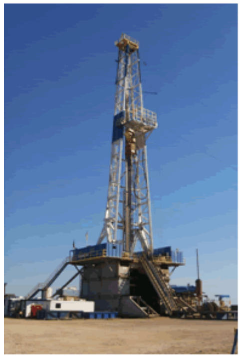
# The Natural Gas Supply Chain





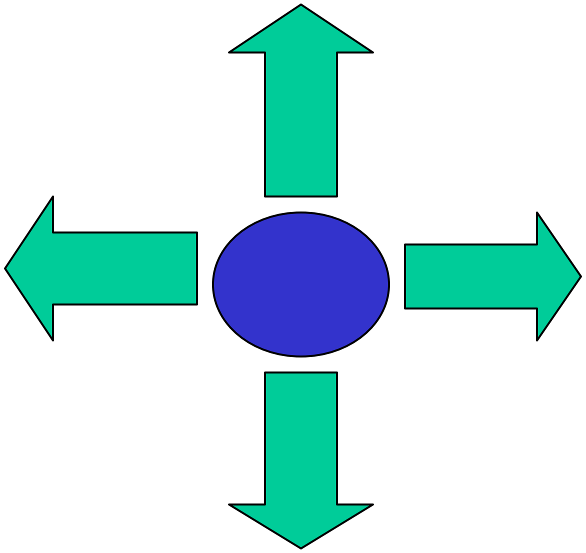
**From well-head to burner-tip**

# Producer's Problem



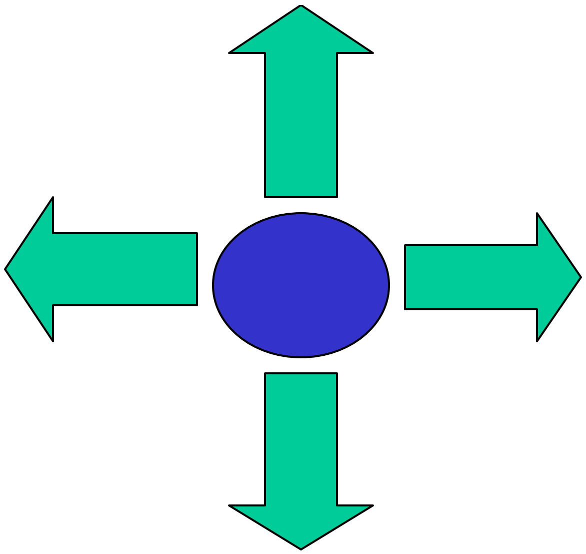

- Maximize production revenues less production costs s.t.
  - bounds on production rates
  - bounds on volume of gas produced in time-window of analysis
- Decision Variables
  - How much to produce in season and year (cubic meters/day)
- Market Clearing
  - Producers' sales must equal Trader's purchases from Producer

# Trader’s Problem



- Maximize selling revenues less purchase costs from domestic producer and neighboring traders
- s.t.
  - material balances, including international pipeline losses
- Decision Variables
  - How much to sell in season and year (cubic meters/day)
  - How much to buy from producers and neighboring transmitters (cubic meters/day)
- Market Clearing:
  - Sales must equal Purchases of (domestic) Marketers, Storage, LNG Liquefaction and (neighboring) Traders

## Trader Characteristics



- Interfaces between producers and end-user markets
- Separate entity
- ‘Dedicated trading companies for each producer’
- Mimics some market aspects better than ‘producer’- ‘marketer’ only
- Allows easier incorporation separate low/high calorific markets

# LNG Liquefier Problem

- Maximize revenues from selling LNG to Regasifiers less purchase, liquefaction and distribution costs s.t.
  - bounds on liquefaction capacity
  - material balance including liquefaction losses
- Decision Variables
  - How much to buy from the Trader
  - How much to sell to each LNG Regasifier
- Market Clearing
  - Sales to a specific Regasifier must equal Purchases by specific Regasifier from this Liquefactor

# LNG Regasifier Problem

- Maximize revenues from selling regasified LNG to marketers and storage less transport and regasifaction costs
- s.t.
  - Regasification capacity
  - Material balance including transport and regasification losses
- Decision Variables
  - How much to sell
  - How much to buy from each liquefactor
- Market Clearing
  - Sales must equal Purchases from this Regasifier by each Marketer and each Storage operator
- (Actually LNG Regasifier operators don't buy and sell gas but Regasification services to marketers. Similar to 'Storage operator')

# Pipeline Operator's Problem

- Maximize congestion revenues

s.t.

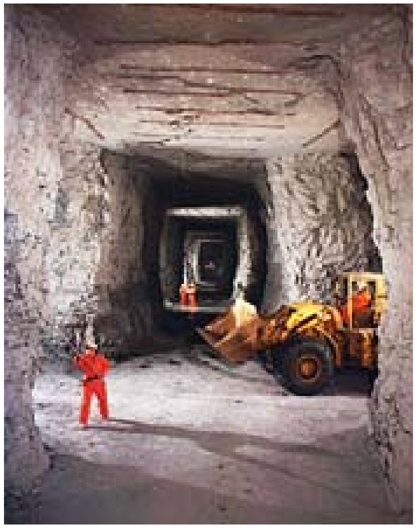
  - capacity bounds on flow

- Decision Variables
  - How much capacity to sell to traders (in each season and year)
- Market Clearing
  - Capacity sold to traders must equal capacity purchased by traders

# Storage Reservoir Operator's Problem

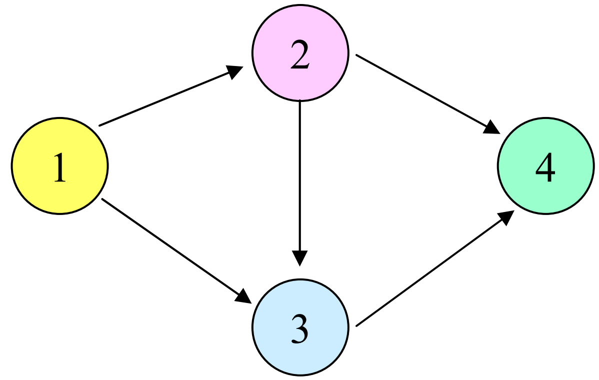
- Maximize net revenues from marketers less injection costs, distribution costs, and purchasing costs from trader and LNG Regasification

  s.t.
  - volumetric bound on working gas
  - maximum extraction rate bound
  - maximum injection rate bound
  - annual injection-extraction balancing



- Decision Variables
  - How much gas to buy from traders and LNG regasifiers
  - How much gas to sell to Marketers
- Market Clearing
  - Storage operators' sales must equal marketers' purchases from storage

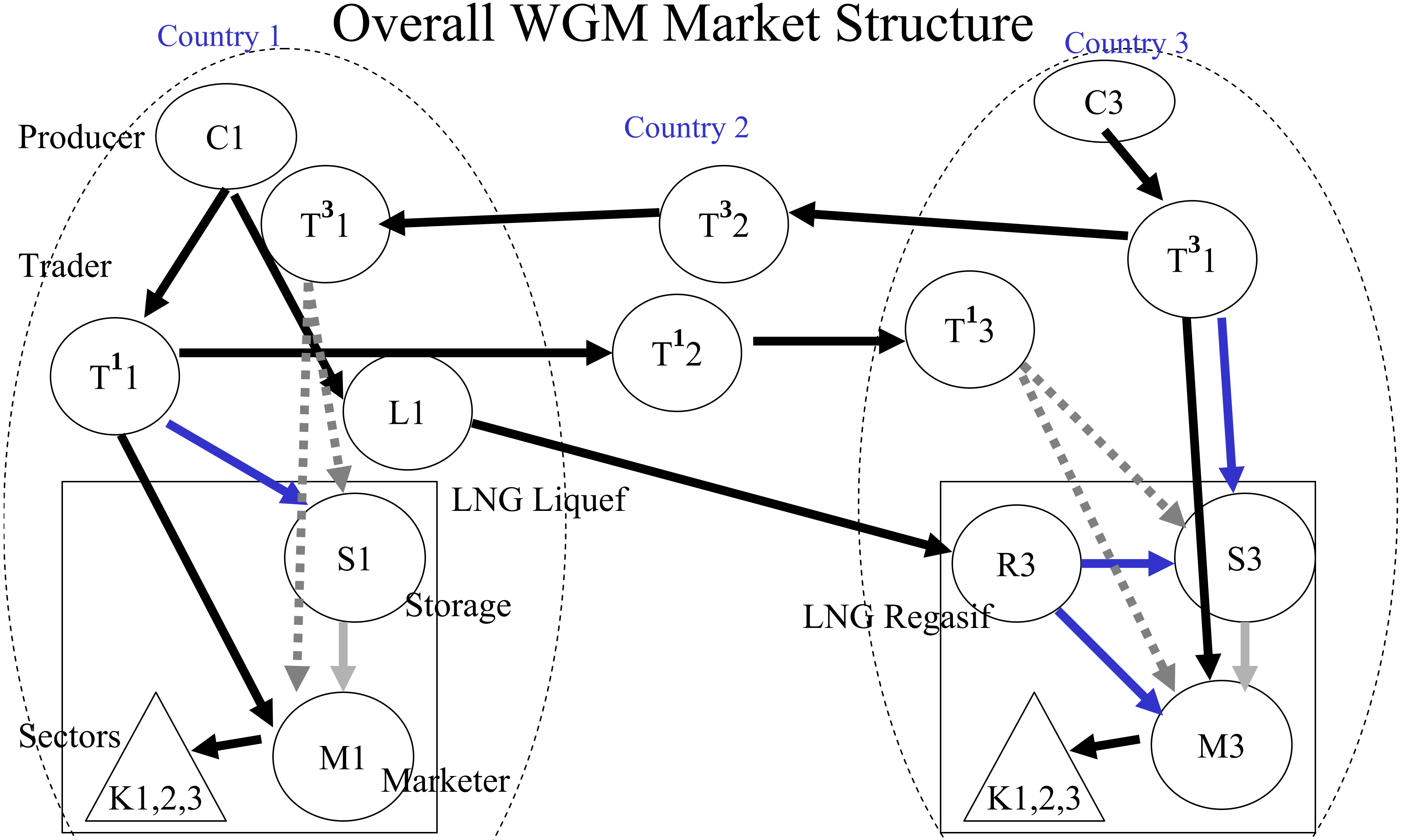
# Marketer/Shipper's Problem





- Maximize demand sector revenues less local delivered costs from transmitter, storage and LNG Regasification
- s.t.
  - Sales to Sectors MUST EQUAL purchases from trader, storage, LNG regasifier
- Decision Variables
  - How much to buy from trader, storage and LNG
  - How much to sell to each sector

# Overall WGM Market Structure




- **Traders are "producer specific contract agents"**
- **Marketers and storage operators can buy from any traders**
- **Liquefier only buys from domestic producer**

# Complementarity Aspects

- Take major players' economic behavior consistent with maximizing net profit subject to economic and engineering constraints (producers, storage operators, pipeline operators, liquefiers, regasifiers, traders)
- Collect all the resulting optimality conditions along with market-clearing ones as well as inverse demand functions representing the consumers
- Resulting set of conditions is a nonlinear complementarity problem (variational inequality)

# World Gas Model

- Countries covered in WGM
  - 73 production/75 consumption
- Typical decision variables
  - operating levels (e.g., production, storage, etc.)
  - investment levels (e.g., pipeline, liquefaction capacity)
- Other
  - LNG contract database not just spot market
  - Multiple years (e.g., 2005, 2010, 2015, 2020, 2030)
  - Computational aspects
    - ~60,000 vars. Solves in 2 hours on a very fast computer (3 GHz, 4GB RAM, 64-bit machine), 2005-2020 timeframe (e.g., 2005, 2010, 2015, 2020)
    - will want to stochasticize the demand (or other components) at some point

# WGM – Production Regions In 2005 70 (+3 in later years)




**Map source: http://en.wikipedia.org/wiki/Wikipedia:Blank_maps**

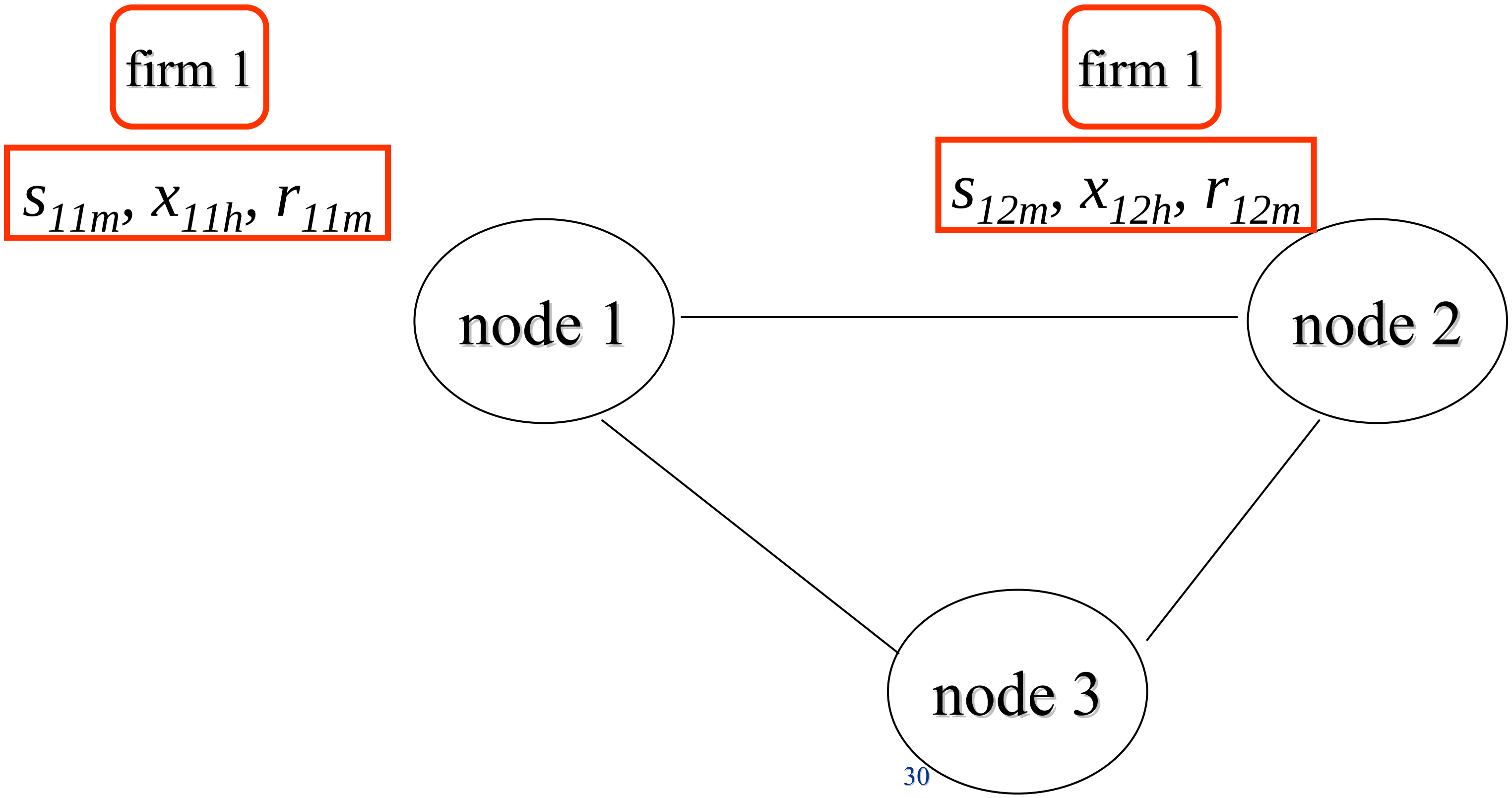
# WGM - Consuming Regions: 75 (non-producing are underlined)




**Map source: http://en.wikipedia.org/wiki/Wikipedia:Blank_maps**

# Stochastic Complementarity Problem for Power Market
# (based on Hobbs (2001) deterministic complementarity problem)

- Firms compete in generation market at each node
- Firms can be at multiple nodes *i*, multiple types of generation units *h*
- Firm *f*'s optimization problem:
  **maximize expected net revenue subject to capacity and consistent constraints**
- Main decision variables are: sales ($s_{fim}$), slow-ramping generation ($x_{fih}$), rapid-ramping generation ($r_{fim}$), *m* relates to scenario with probability *prob(i,m)*



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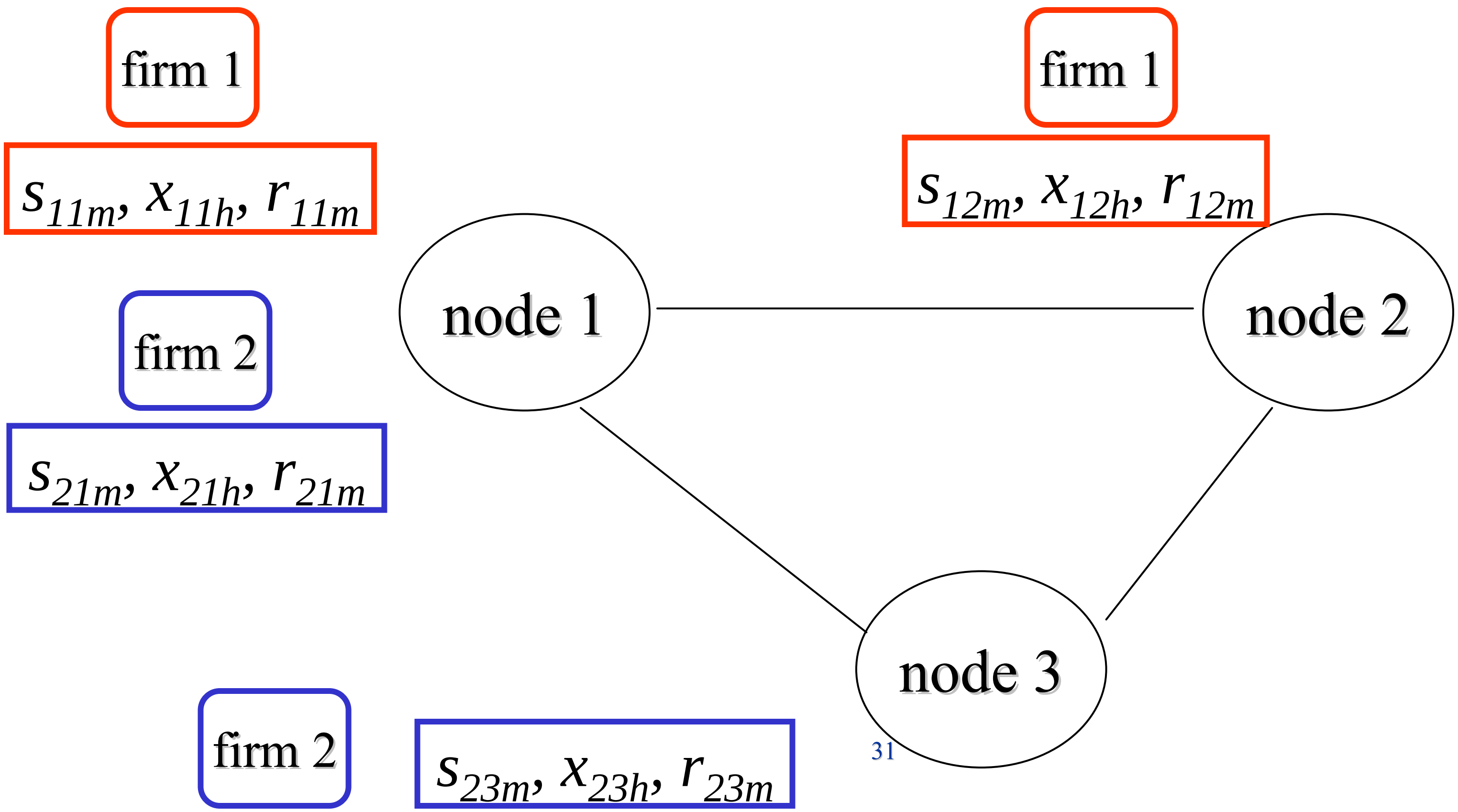

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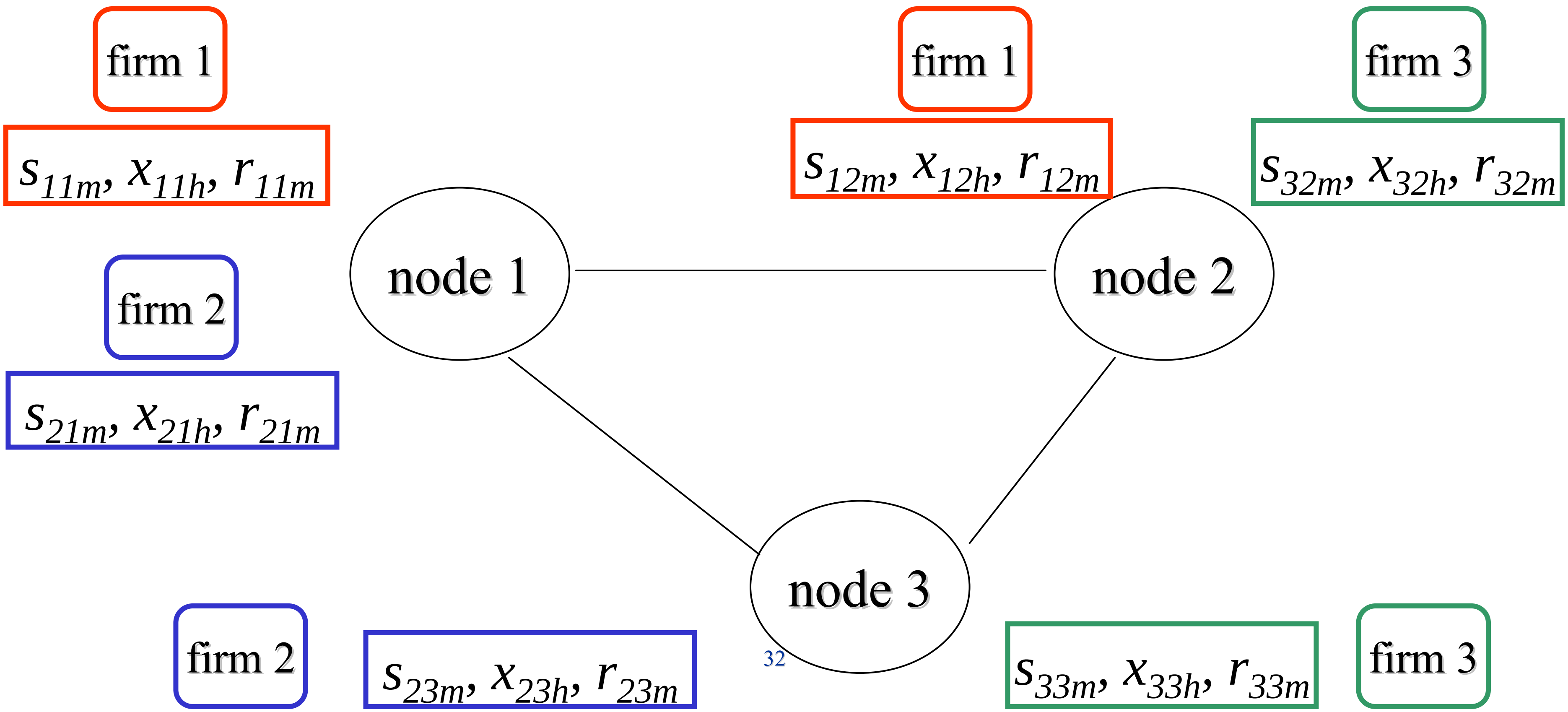


**inverse demand**

$$\max_{s_f, x_f, r_f} \left[ \begin{array}{c} \sum_m \sum_i prob(i,m) \left( \boxed{a_{im} - b_i \left( \sum_g s_{gim} \right)} - w_{im} \right) s_{fim} \\ - \sum_m \sum_{i,h} prob(i,m) \left( C_{fih} - w_{im} \right) x_{fih} - \sum_m \sum_i prob(i,m) \left( RC_{fi} - w_{im} \right) r_{fim} \end{array} \right] \quad (1a)$$

$$s.t.\ x_{fih} - X_{fih} \leq 0, \forall f, i, h \qquad (\rho_{fih})\ (1b)$$

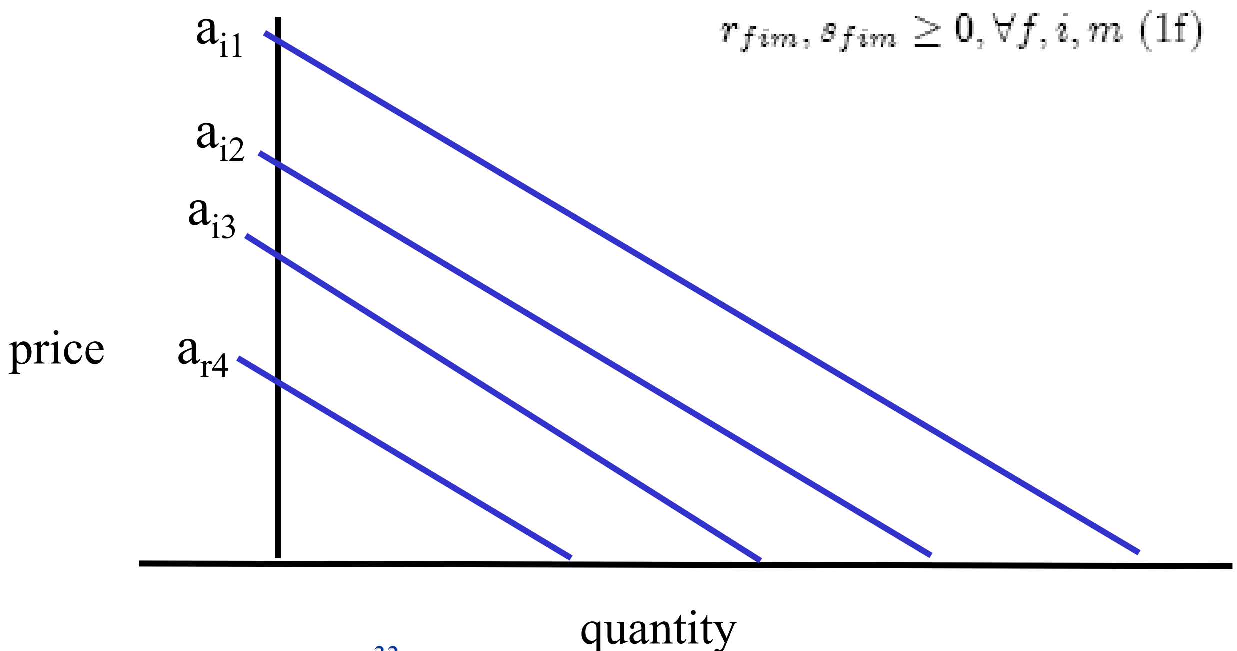
$$r_{fim} - R_{fi} \leq 0\ \forall f, i, m \qquad (\sigma_{fim})\ (1c)$$

$$\sum_i s_{fim} - \sum_{i,h} x_{fih} - \sum_i r_{fim} = 0, \forall f, m\ (\theta_{fm})\ (1d)$$

$$x_{fih} \geq 0, \forall f, i, h\ (1e)$$

$$r_{fim}, s_{fim} \geq 0, \forall f, i, m\ (1f)$$

- Inverse demand intercept is random
- pmf for intercept, slope certain



**wheeling fee**

$$\max_{s_f, x_f, r_f} \left[ \begin{array}{c} \sum_m \sum_i prob(i,m) \left( a_{im} - b_i \left( \sum_g s_{gim} \right) - \boxed{w_{im}} \right) s_{fim} \\ - \sum_m \sum_{i,h} prob(i,m) \left( C_{fih} - w_{im} \right) x_{fih} - \sum_m \sum_i prob(i,m) \left( RC_{fi} - w_{im} \right) r_{fim} \end{array} \right] \quad (1a)$$

$$s.t. \; x_{fih} - X_{fih} \leq 0, \forall f, i, h \qquad (\rho_{fih}) \quad (1b)$$

$$r_{fim} - R_{fi} \leq 0 \; \forall f, i, m \qquad (\sigma_{fim}) \quad (1c)$$

$$\sum_i s_{fim} - \sum_{i,h} x_{fih} - \sum_i r_{fim} = 0, \forall f, m \; (\theta_{fm}) \quad (1d)$$

$$x_{fih} \geq 0, \forall f, i, h \quad (1e)$$

$$r_{fim}, s_{fim} \geq 0, \forall f, i, m \quad (1f)$$

**expected revenue**

$$\max_{s_f, x_f, r_f} \left[ \begin{array}{c} \sum_m \sum_i prob(i,m) \left( a_{im} - b_i \left( \sum_g s_{gim} \right) - w_{im} \right) s_{fim} \\ - \sum_m \sum_{i,h} prob(i,m) \left( C_{fih} - w_{im} \right) x_{fih} - \sum_m \sum_i prob(i,m) \left( RC_{fi} - w_{im} \right) r_{fim} \end{array} \right] \quad (1a)$$

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$$\max_{s_f, x_f, r_f} \left[ \begin{array}{c} \sum_m \sum_i prob(i,m) \left( a_{im} - b_i \left( \sum_g s_{gim} \right) - w_{im} \right) s_{fim} \\ - \sum_m \sum_{i,h} prob(i,m) \left( C_{fih} - w_{im} \right) x_{fih} - \sum_m \sum_i prob(i,m) \left( RC_{fi} - w_{im} \right) r_{fim} \end{array} \right] \quad (1a)$$

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$$x_{fih} \geq 0, \forall f, i, h \ (1e)$$

$$r_{fim}, s_{fim} \geq 0, \forall f, i, m \ (1f)$$

**expected slow-ramping generation costs**

$$\max_{s_f, x_f, r_f} \left[ \begin{array}{c} \sum_m \sum_i prob(i,m) \left( a_{im} - b_i \left( \sum_g s_{gim} \right) - w_{im} \right) s_{fim} \\ - \sum_m \sum_{i,h} prob(i,m) \left( C_{fih} - w_{im} \right) x_{fih} - \boxed{\sum_m \sum_i prob(i,m) \left( RC_{fi} - w_{im} \right) r_{fim}} \end{array} \right] \quad (1a)$$

$$s.t.\ x_{fih} - X_{fih} \leq 0, \forall f, i, h \qquad (\rho_{fih}) \quad (1b)$$

$$r_{fim} - R_{fi} \leq 0\ \forall f, i, m \qquad (\sigma_{fim}) \quad (1c)$$

$$\sum_i s_{fim} - \sum_{i,h} x_{fih} - \sum_i r_{fim} = 0, \forall f, m \ (\theta_{fm}) \quad (1d)$$

$$x_{fih} \geq 0, \forall f, i, h \quad (1e)$$

$$r_{fim}, s_{fim} \geq 0, \forall f, i, m \quad (1f)$$

**expected rapid-ramping generation costs**

$$\max_{s_f, x_f, r_f} \left[ \begin{array}{c} \sum_m \sum_i prob(i,m) \left( a_{im} - b_i \left( \sum_g s_{gim} \right) - w_{im} \right) s_{fim} \\ - \sum_m \sum_{i,h} prob(i,m) \left( C_{fih} - w_{im} \right) x_{fih} - \sum_m \sum_i prob(i,m) \left( RC_{fi} - w_{im} \right) r_{fim} \end{array} \right] \quad (1a)$$

$$s.t. \; x_{fih} - X_{fih} \leq 0, \forall f, i, h \quad (\rho_{fih}) \; (1b)$$

$$r_{fim} - R_{fi} \leq 0 \; \forall f, i, m \quad (\sigma_{fim}) \; (1c)$$

**capacity constraints**

$$\sum_i s_{fim} - \sum_{i,h} x_{fih} - \sum_i r_{fim} = 0, \forall f, m \; (\theta_{fm}) \; (1d)$$

$$x_{fih} \geq 0, \forall f, i, h \; (1e)$$

$$r_{fim}, s_{fim} \geq 0, \forall f, i, m \; (1f)$$

$$\max_{s_f, x_f, r_f} \left[ \begin{array}{c} \sum_m \sum_i prob(i,m) \left( a_{im} - b_i \left( \sum_g s_{gim} \right) - w_{im} \right) s_{fim} \\ - \sum_m \sum_{i,h} prob(i,m) \left( C_{fih} - w_{im} \right) x_{fih} - \sum_m \sum_i prob(i,m) \left( RC_{fi} - w_{im} \right) r_{fim} \end{array} \right] \quad (1a)$$

$$s.t.\ x_{fih} - X_{fih} \leq 0, \forall f, i, h \qquad \left( \rho_{fih} \right) \quad (1b)$$

$$r_{fim} - R_{fi} \leq 0\ \forall f, i, m \qquad \left( \sigma_{fim} \right) \quad (1c)$$

**consistency constraints**

**sales= generation**

$$\boxed{\sum_i s_{fim} - \sum_{i,h} x_{fih} - \sum_i r_{fim} = 0} \ \forall f, m \ \left( \theta_{fm} \right) \quad (1d)$$

$$x_{fih} \geq 0, \forall f, i, h \quad (1e)$$

$$r_{fim}, s_{fim} \geq 0, \forall f, i, m \quad (1f)$$

# KKT Conditions for Firm *f*'s Problem

$$0 \le prob(i,m)\left[-a_{im} + b_i\left(s_{fim+}\sum_g s_{gim}\right) + w_{im}\right] \tag{2a}$$

$$+\theta_{fm} \perp s_{fim} \ge 0, \forall f, i, m$$

$$0 \le C_{fih} - \sum_m prob(i,m)w_{im} - \sum_m \theta_{fm} + \rho_{fih} \perp x_{fih} \ge 0 \tag{2b}$$

$$\forall f, i, h$$

$$0 \le prob(i,m)RC_{fi} + \sigma_{fim} - prob(i,m)w_{im} - \theta_{fm} \perp r_{fim} \ge 0 \tag{2c}$$

$$\forall f, i, m$$

$$0 \le X_{fih} - x_{fih} \perp \rho_{fih} \ge 0 \tag{2d}$$

$$\forall f, i, h$$

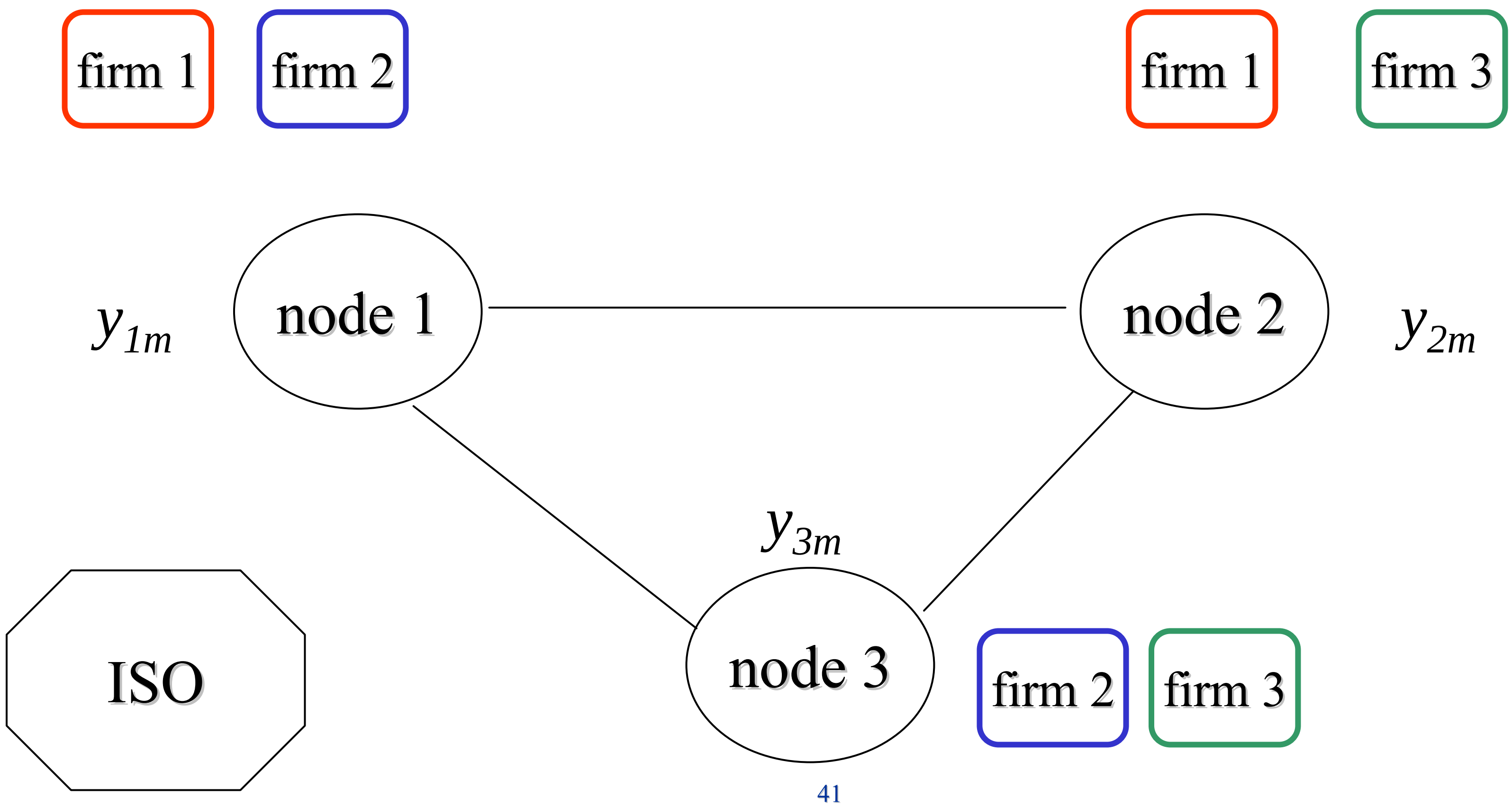
$$0 \le R_{fi} - r_{fim} \perp \sigma_{fim} \ge 0 \tag{2e}$$

$$\forall f, i, m$$

$$0 = \sum_i s_{fim} - \sum_{i,h} x_{fih} - \sum_i r_{fim}, \ \theta_{fm} \text{ free} \tag{2f}$$

$$\forall f, m$$

- Independent System Operator (ISO) manages the grid
- Optimization problem:
  **maximize expected wheeling fees subject to consistency constraints for line flows**
- Main variables are: $y_{im}$ , if positive then inflow, if negative then outflow at node *i*



$$\max_{y} \sum_{m} \sum_{i} prob(i,m) w_{im} y_{im} \tag{3a}$$

expected wheeling fees

$$s.t. \ -T_{l-} - \sum_{i} PTDF_{il} y_{im} \leq 0, \forall l, m \quad (\lambda_{lm-}) \tag{3b}$$

$$-T_{l+} + \sum_{i} PTDF_{il} y_{im} \leq 0, \forall l, m \ (\lambda_{lm+}) \tag{3c}$$

$$\max_{y} \sum_{m} \sum_{i} prob(i,m) w_{im} y_{im} \tag{3a}$$

$$s.t. \quad -T_{l-} - \sum_{i} PTDF_{il} y_{im} \leq 0, \forall l, m \quad (\lambda_{lm-}) \tag{3b}$$

$$-T_{l+} + \sum_{i} PTDF_{il} y_{im} \leq 0, \forall l, m \quad (\lambda_{lm+}) \tag{3c}$$

**line limit constraints**

# KKT Conditions for ISO's Problem

$$0 = -prob(i,m)w_{im} - \sum_l PTDF_{il}\lambda_{lm-} + \sum_l PTDF_{il}\lambda_{lm+}, \quad y_{im} \text{ free} \quad \forall i,m \tag{4a}$$

$$0 \leq T_{l-} + \sum_i PTDF_{il}y_{im} \perp \lambda_{lm-} \geq 0 \quad \forall l,m \tag{4b}$$

$$0 \leq T_{l+} - \sum_i PTDF_{il}y_{im} \perp \lambda_{lm+} \geq 0 \quad \forall l,m \tag{4c}$$

- Market-clearing constraints
- Balance sales, generation, and flows at each node

sales

$$0 = \boxed{-\sum_f s_{fim}} + \sum_{f,h} x_{fih} + \sum_f r_{fim} + y_{im} \text{ with free dual variable } \hat{w}_{im} \equiv prob(i,m) w_{im} \quad (5)$$

- Overall stochastic linear complementarity problem (LCP) is (2), (4), (5)
- Question: Why not just solve all these conditions together, i.e., extensive form of the stochastic LCP?
- Answer: May have many scenarios *m* and this would make it an especially large LCP which could be computationally prohibitive. Instead can use Benders Decomposition

- Market-clearing constraints
- Balance sales, generation, and flows at each node

generation

$$0 = -\sum_{f} s_{fim} + \boxed{\sum_{f,h} x_{fih} + \sum_{f} r_{fim}} + y_{im} \text{ with free dual variable } \hat{w}_{im} \equiv prob(i,m) w_{im} \quad (5)$$

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- Market-clearing constraints
- Balance sales, generation, and flows at each node

**inflows/outflows**

$$0 = -\sum_f s_{fim} + \sum_{f,h} x_{fih} + \sum_f r_{fim} + \boxed{y_{im}} \text{ with free dual variable } \hat{w}_{im} \equiv prob(i,m) w_{im} \quad (5)$$

- Overall stochastic linear complementarity problem (LCP) is (2), (4), (5)
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- Market-clearing constraints
- Balance sales, generation, and flows at each node

wheeling fees

$$0 = -\sum_{f} s_{fim} + \sum_{f,h} x_{fih} + \sum_{f} r_{fim} + y_{im} \text{ with free dual variable } \hat{w}_{im} \equiv prob(i,m) w_{im} \quad (5)$$

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- **Question**: Why not just solve all these conditions together, i.e., extensive form of the stochastic LCP?

- **Answer**: May have many scenarios *m* and this would make it an especially large LCP which could be computationally prohibitive
- We use a Benders-like method adapted from Fuller and Chung (2007) to decompose the problem appropriately
- Master problem (MP) will have variables independent of scenarios (e.g., $x_{fih}$)
- Suproblem (SP) will have scenario-dependent variables and can be solved separately by scenario (or not)
- Will apply a Dantzig-Wolfe method for VIs (Fuller and Chung) to the "dual VI" of our problem resulting in a Benders-like method for stochastic (linear) complementarity problems

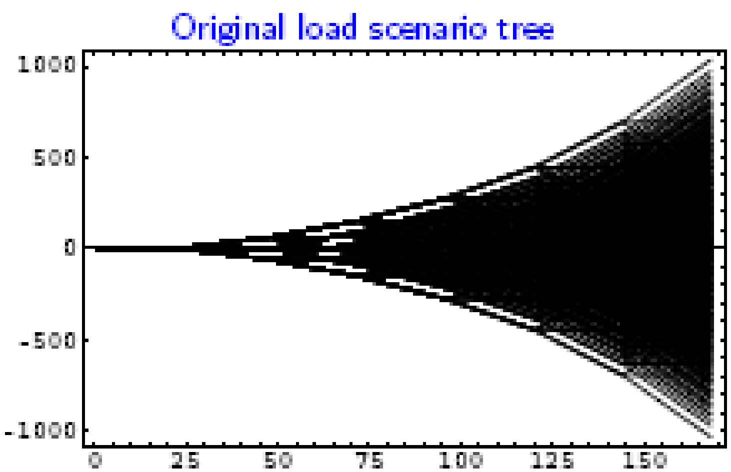
# Algorithms
# Benders for Stochastic Complementarity Problems (results to be shown)

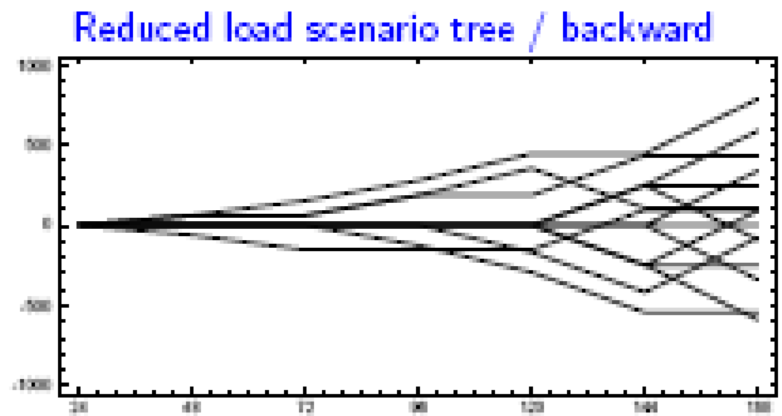
# Scenario Reduction Methods

# Scenario Reduction Methods

# Stochastic Optimization Background

- Many attempts to solve such a stochastic problem, some examples of approaches
  - Decomposing the problem (e.g., L-shaped method)
  - Using a sampling approach
  - Using a scenario tree for the finite (but usually large) number of realizations, then approximating it with a reduced tree



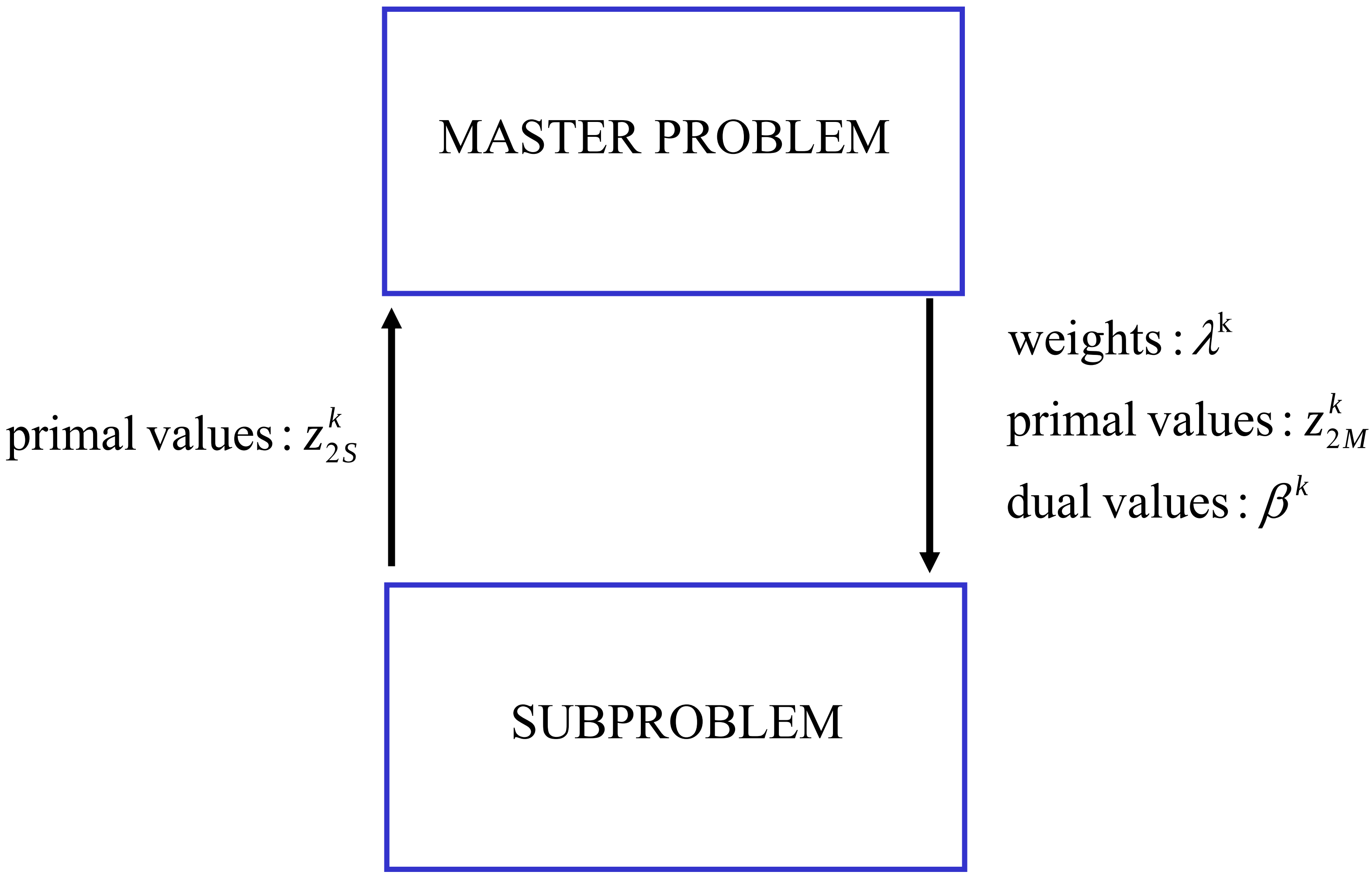




Römisch, Dupačová, Gröwe-Kuska, Heitsch (2003)

# Stochastic Optimization Background

- We came up with a new merit function for stochastic VI's that helps to decide when the reduced tree is "good enough"
- For details see:
  **S.A. Gabriel**, J. Zhuang, R. Egging, "Solving Stochastic Complementarity Problems in Energy Market Modeling Using Scenario Reduction," *European Journal of Operational Research*, December 2007, accepted.

# Overall Benders Approach



# Selected Benders Method Numerical Results

# Selected Numerical Results

- Three discrete probability distributions tried:
  - Symmetric
  - Right-skewed
  - uniform
- Varying number of scenarios
  - 1000, 2000, 3000, 5000, 10000
- No special computations for subproblem (i.e., splitting up into separate problems), although GAMS may be doing some of this on its own

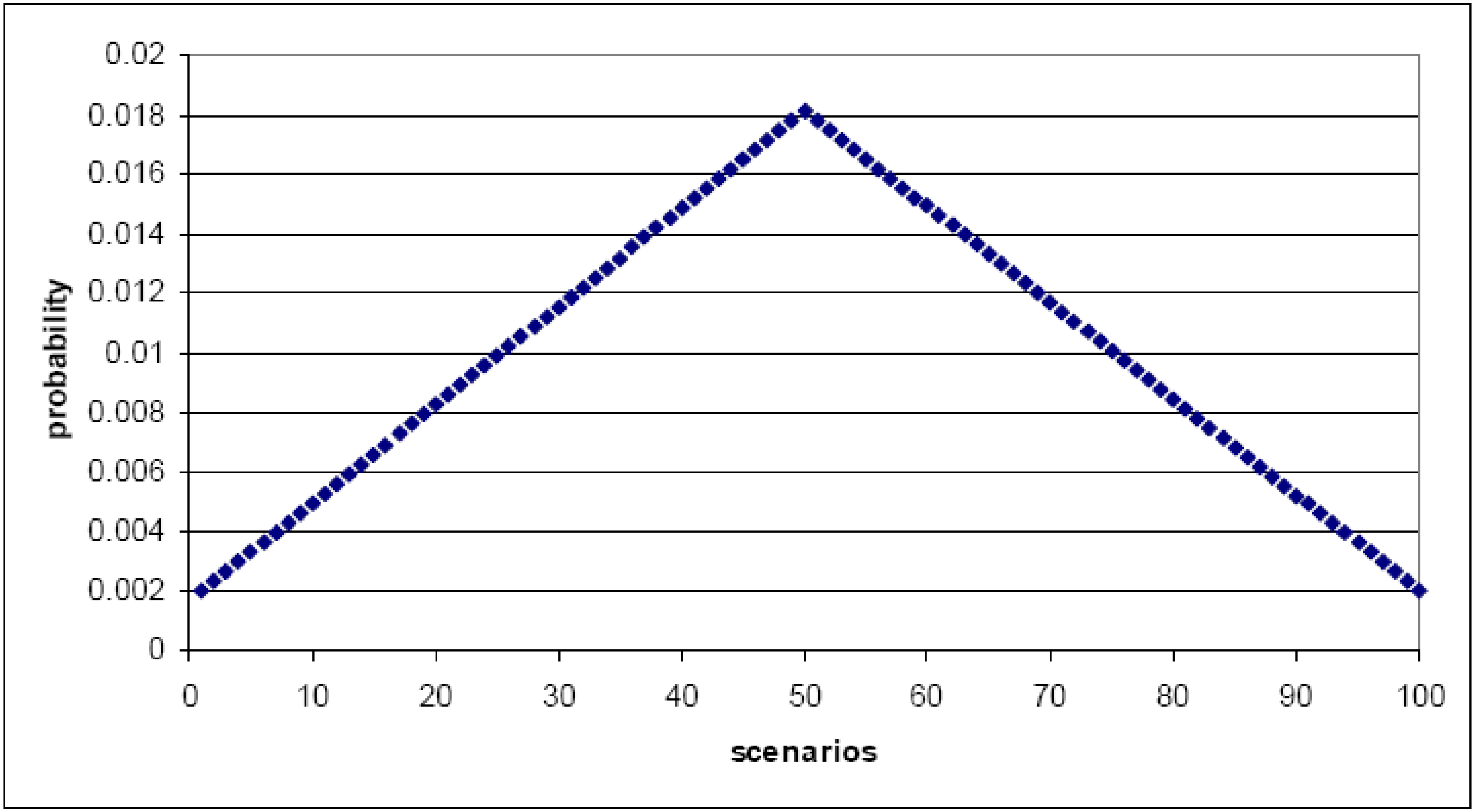



Figure 1: Discrete Symmetric, Triangular Dist. ($\alpha = 0.1, \nu = 0.9, \omega = 0.1, m_1 = \left\lfloor \frac{|M|}{2} \right\rfloor, |M| = 100$)

# of
scenarios

| $\|M\|$ | # of Vars Benders:Ext. Form | Benders Time (SP+ MP= Total) (s) | Ext. Form Time (s) | # of Benders Iters. | Benders Time/Ext. Form Time (%) |
|---|---|---|---|---|---|
| 1,000 | 28,000:28,016 | 2.34+0.12=2.46 | 6.97 | 4 | 35% |
| 2,000 | 56,000:56,016 | 8.05+0.19=8.24 | 36.17 | 4 | 23% |
| 3,000 | 84,000:84,016 | 17.80+0.19=17.99 | 103.38 | 4 | 17% |
| 5,000 | 140,000:140,016 | 57.17+0.19=57.36 | 453.97 | 4 | 13% |
| 10,000 | 280,000:280,016 | 294.08+0.17=294.25 | 3398.67 | 4 | 9% |

Table 8: Numerical Results: $\alpha = 0.1, \nu = 0.9, \omega = 0.1$, start=0.9,end=1.5, Symmetric, Discrete, Triangular Distribution.

**# of Variables**

| $\lvert M \rvert$ | # of Vars Benders:Ext. Form | Benders Time (SP+ MP= Total) (s) | Ext. Form Time (s) | # of Benders Iters. | Benders Time/Ext. Form Time (%) |
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Table 8: Numerical Results: $\alpha = 0.1, \nu = 0.9, \omega = 0.1$, start=0.9,end=1.5, Symmetric, Discrete, Triangular Distribution.

## Computational time for subproblem (SP) and master problem (MP) using Benders-like decomposition

| $\|M\|$ | # of Vars Benders:Ext. Form | Benders Time (SP+ MP= Total) (s) | Ext. Form Time (s) | # of Benders Iters. | Benders Time/Ext. Form Time (%) |
|---|---|---|---|---|---|
| 1,000 | 28,000:28,016 | 2.34+0.12=2.46 | 6.97 | 4 | 35% |
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Table 8: Numerical Results: $\alpha = 0.1, \nu = 0.9, \omega = 0.1$, start=0.9,end=1.5, Symmetric, Discrete, Triangular Distribution.

**Computational time for extensive form (no decomposition)**

| $\|M\|$ | # of Vars Benders:Ext. Form | Benders Time (SP+ MP= Total) (s) | Ext. Form Time (s) | # of Benders Iters. | Benders Time/Ext. Form Time (%) |
|---|---|---|---|---|---|
| 1,000 | 28,000:28,016 | 2.34+0.12=2.46 | 6.97 | 4 | 35% |
| 2,000 | 56,000:56,016 | 8.05+0.19=8.24 | 36.17 | 4 | 23% |
| 3,000 | 84,000:84,016 | 17.80+0.19=17.99 | 103.38 | 4 | 17% |
| 5,000 | 140,000:140,016 | 57.17+0.19=57.36 | 453.97 | 4 | 13% |
| 10,000 | 280,000:280,016 | 294.08+0.17=294.25 | 3398.67 | 4 | 9% |

Table 8: Numerical Results: $\alpha = 0.1, \nu = 0.9, \omega = 0.1$, start=0.9,end=1.5, Symmetric, Discrete, Triangular Distribution.

# of Benders Iterations

| $\|M\|$ | # of Vars Benders:Ext. Form | Benders Time (SP+ MP= Total) (s) | Ext. Form Time (s) | # of Benders Iters. | Benders Time/Ext. Form Time (%) |
|---|---|---|---|---|---|
| 1,000 | 28,000:28,016 | 2.34+0.12=2.46 | 6.97 | 4 | 35% |
| 2,000 | 56,000:56,016 | 8.05+0.19=8.24 | 36.17 | 4 | 23% |
| 3,000 | 84,000:84,016 | 17.80+0.19=17.99 | 103.38 | 4 | 17% |
| 5,000 | 140,000:140,016 | 57.17+0.19=57.36 | 453.97 | 4 | 13% |
| 10,000 | 280,000:280,016 | 294.08+0.17=294.25 | 3398.67 | 4 | 9% |

Table 8: Numerical Results: $\alpha = 0.1, \nu = 0.9, \omega = 0.1$, start=0.9,end=1.5, Symmetric, Discrete, Triangular Distribution.

Decomposed Approach Time/Extensive Form Time

| $\|M\|$ | # of Vars Benders:Ext. Form | Benders Time (SP+ MP= Total) (s) | Ext. Form Time (s) | # of Benders Iters. | Benders Time/Ext. Form Time (%) |
|---|---|---|---|---|---|
| 1,000 | 28,000:28,016 | 2.34+0.12=2.46 | 6.97 | 4 | 35% |
| 2,000 | 56,000:56,016 | 8.05+0.19=8.24 | 36.17 | 4 | 23% |
| 3,000 | 84,000:84,016 | 17.80+0.19=17.99 | 103.38 | 4 | 17% |
| 5,000 | 140,000:140,016 | 57.17+0.19=57.36 | 453.97 | 4 | 13% |
| 10,000 | 280,000:280,016 | 294.08+0.17=294.25 | 3398.67 | 4 | 9% |

Table 8: Numerical Results: $\alpha = 0.1, \nu = 0.9, \omega = 0.1$, start=0.9,end=1.5, Symmetric, Discrete, Triangular Distribution.

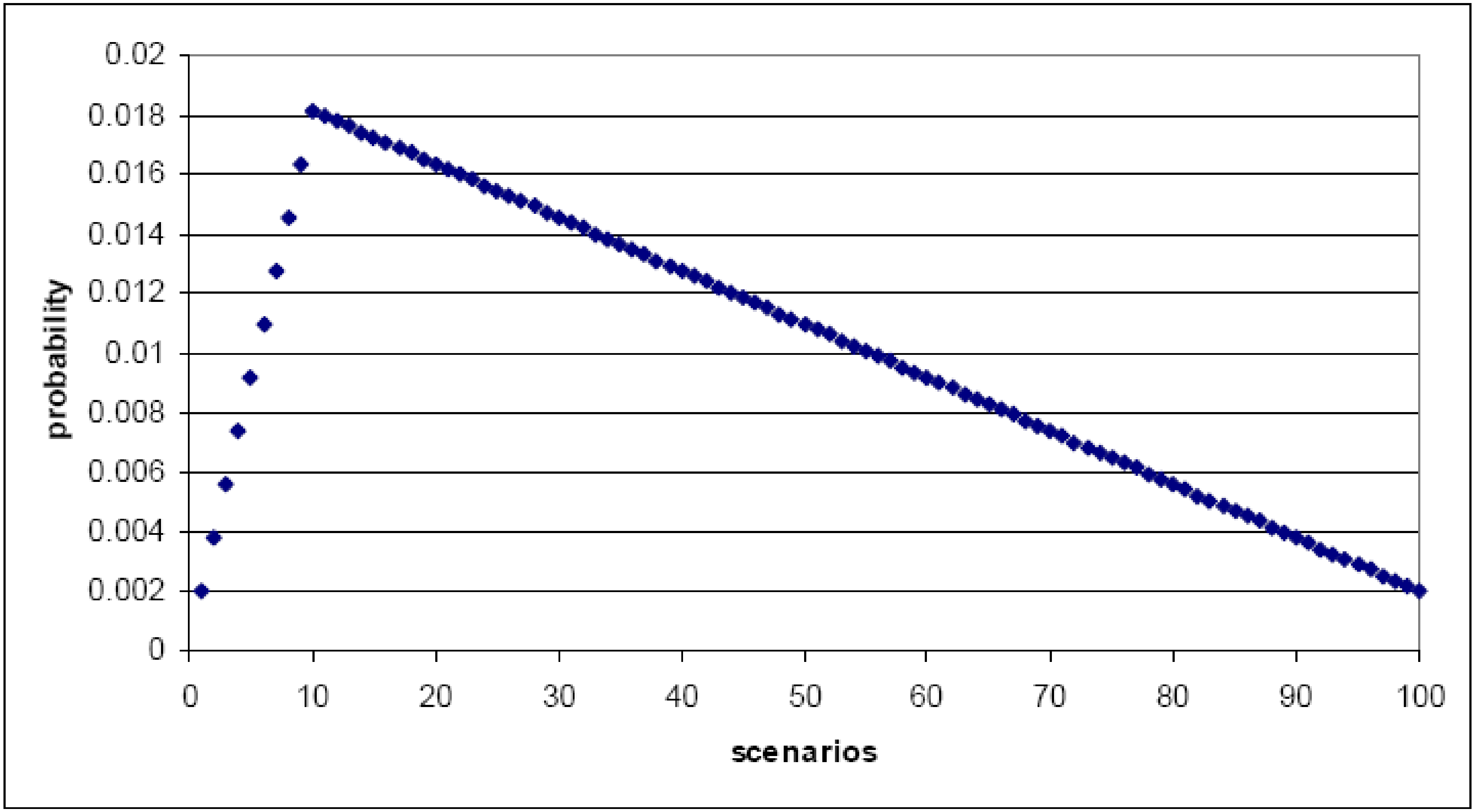


Figure 2: Discrete Right-Skewed, Triangular Dist. ($\alpha = 0.1, \nu = 0.9, \omega = 0.1, m_1 = \left\lfloor \frac{|M|}{10} \right\rfloor, |M| = 100$)

| $\|M\|$ | # | Benders | Ext. | # | Benders |
|---|---|---|---|---|---|
| | of | Time | Form | of | Time/Ext. |
| | Vars<br>Benders:Ext. Form | (SP+ MP= Total)<br>(s) | Time<br>(s) | Benders<br>Iters. | Form Time<br>(%) |
| 1,000 | 28,000:28,016 | 3.84+0.22=4.06 | 7.58 | 6 | 54% |
| 2,000 | 56,000:56,016 | 11.67+0.42=12.09 | 40.56 | 6 | 30% |
| 3,000 | 84,000:84,016 | 40.23+0.33=40.56 | 118.70 | 6 | 34% |
| 5,000 | 140,000:140,016 | 106.22+0.26=106.48 | 504.53 | 6 | 21% |
| 10,000 | 280,000:280,016 | 382.98+0.25=383.23 | 4042.25 | 6 | 9% |

Table 9: Numerical Results: $\alpha = 0.1, \nu = 0.9, \omega = 0.1$, start=0.9,end=1.5, Right-Skewed, Discrete, Triangular Distribution

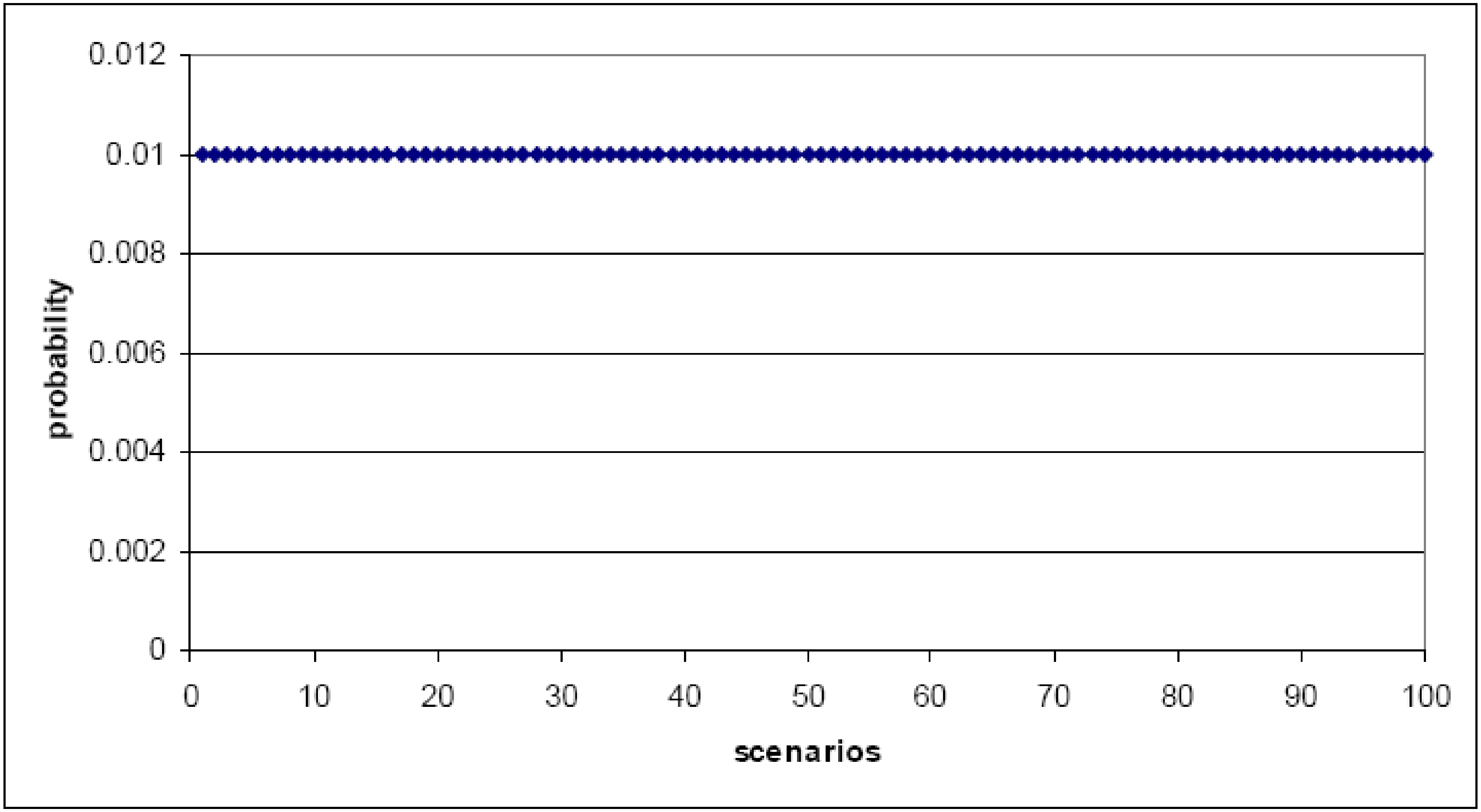


Figure 3: Discrete Uniform Dist. $(\alpha = 0.1, \nu = 0.1, \omega = 0.1, m_1 = \left\lfloor \frac{|M|}{2} \right\rfloor, |M| = 100)$

| $\|M\|$ | # of Vars Benders:Ext. Form | Benders Time (SP+ MP= Total) (s) | Ext. Form Time (s) | # of Benders Iters. | Benders Time/Ext. Form Time (%) |
|---|---|---|---|---|---|
| 1,000 | 28,000:28,016 | 2.28+0.30=2.58 | 6.86 | 4 | 38% |
| 2,000 | 56,000:56,016 | 6.98+0.16=7.14 | 37.02 | 4 | 19% |
| 3,000 | 84,000:84,016 | 25.92+0.19=26.11 | 106.69 | 4 | 24% |
| 5,000 | 140,000:140,016 | 69.58+0.05=69.63 | 431.27 | 4 | 16% |
| 10,000 | 280,000:280,016 | 250.06+0.19=250.25 | 3439.97 | 4 | 7% |

Table 12: Numerical Results: $\alpha = 0.1, \nu = 0.1, \omega = 0.1$, start=0.9,end=1.5, Uniform, Triangular Distribution

# Ongoing Work

- Apply Benders-like decomposition method to stochastic complementarity problems for natural gas using World Gas Model (WGM), WGM has multiple years, seasons, players, and over 70 countries

# Related Publications


1. S. A. Gabriel, S. Vikas, D. M. Ribar, 2000. "Measuring the Influence of Canadian Carbon Stabilization Programs on Natural Gas Exports to the United States via a Bottom-Up Intertemporal Spatial Price Equilibrium Model," *Energy Economics*, 22, 497-525.
2. S. A. Gabriel, J. Manik , S. Vikas, 2003. "Computational Experience with a Large-Scale, Multi-Period, Spatial Equilibrium Model of the North American Natural Gas System," *Networks and Spatial Economics*,3, 97-122.
3. S. A. Gabriel, S. Kiet, J. Zhuang, 2005. "A Mixed Complementarity-Based Equilibrium Model of Natural Gas Markets", *Operations Research*, 53(5), 799-818.
4. S. A. Gabriel, J. Zhuang, S. Kiet, 2005. "A Large-Scale Complementarity Model of the North American Natural Gas Market", *Energy Economics*, 27, 639-665.
5. R. Egging, S. A. Gabriel, "Examining Market Power in the European Natural Gas Market", 2006. *Energy Policy*, 34 (17), 2762-2778.
6. S. A. Gabriel and Y. Smeers, 2006. "Complemenatarity Problems in Restructured Natural Gas Markets," *Recent Advances in Optimization. Lecture Notes in Economics and Mathematical Systems, Edited by A. Seeger*,Vol. 563, Springer-Verlag Berlin Heidelberg, 343-373.
7. J. Zhuang, S.A. Gabriel, 2008, "A Complementarity Model for Solving Stochastic Natural Gas Market Equilibria," *Energy Economics 30*(1), 113-147.
8. S.A. Gabriel, J. Zhuang, R. Egging, “Solving Stochastic Complementarity Problems in Energy Market Modeling Using Scenario Reduction,” *European Journal of Operational Research*, December 2007, forthcoming.
9. R. Egging, S.A. Gabriel, F.Holz, J. Zhuang, "A Complementarity Model for the European Natural Gas Market," *Energy Policy*, January, 2008, forthcoming.