NODAL PRICES IN THE DAY-AHEAD MARKET

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What we cover

• Two-stage stochastic program for contingency analysis in the day-ahead auction.
• Find the LMPs and the expected marginal value of electricity from the dual variables.
• Show differences with current duals
• Show marginal value problem
• Bouffard, Galiana, and Conejo (2005)
Objective function:

$$\max_{P_{gi}, L_i, R_i, S_i} \left( \sum_{i=1}^{N} \sum_{l=1}^{nb} \sum_{i=1}^{nq} C_{Li} L_i^l - \sum_{i=1}^{N} \sum_{q=1}^{nob} C_{qi} P_{gi} \right) + \sum_{k=1}^{K} \sum_{i=1}^{N} \left( \sum_{u=1}^{ns} b_{giu} S_{iu}^k - \sum_{v=1}^{nr} b_{Li}^{v} R_i^{v} \right)$$

Inequality constraints for offer/bid blocks and total node generation/load bounds, respectively:

$$P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max} \quad L_i^{l, min} \leq L_i^l \leq L_i^{l, max}$$

$$P_{gi} = \sum_{q=1}^{no} P_{gi}^q \quad \lambda_{gi} \quad L_i = \sum_{l=1}^{nb} L_i^l \quad \lambda_{Li}$$

$$P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max} \quad L_i^{l, min} \leq L_i^l \leq L_i^{l, max}$$

Power-balance equality constraint and branch-flow inequality constraints for normal operating conditions.

$$NG_i = P_{gi} - L_i \quad \lambda_{NGi} \quad \sum_{i=1}^{n} NG_i = 0 \quad \lambda_{bal}$$

$$-P_{bj}^{max} \leq \sum_{i=1}^{n} SF_{ji} \cdot NG_i \leq P_{bj}^{max}$$
Generation reduction and load-shedding inequality constraints, 
load balance equality constraints and branch flow inequality 
constraints for contingency conditions

\[ 0 \leq S_{i}^{u,k} \leq S_{i}^{u,k,\text{max}} \quad 0 \leq R_{i}^{v,k} \leq R_{i}^{v,k,\text{max}} \]

\[ \sum_{u=1}^{n_{S}} S_{i}^{u,k} + P_{gi}^{k} = P_{gi} \quad \gamma_{gi}^{k} \quad \sum_{v=1}^{n_{r}} R_{i}^{v,k} + L_{i}^{k} = L_{i} \quad \gamma_{Li}^{k} \]

\[ N_{Gi}^{k} = P_{gi}^{k} - L_{i}^{k} \quad \gamma_{NGi}^{k} \]

\[ \sum_{i=1}^{N} N_{Gi}^{k} = 0 \quad \gamma_{bal}^{k} \]

\[ -P_{bj}^{k,\text{max}} \leq \sum_{i=1}^{n} S_{fj}^{k} \cdot N_{Gi}^{k} \leq P_{bj}^{k,\text{max}} \quad \mu_{j}^{k} \]

<table>
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<th>( L_{i} )</th>
<th>( P_{gi}^{q} )</th>
<th>( P_{gi} )</th>
<th>( L_{i}^{k} )</th>
<th>( P_{gi}^{k} )</th>
<th>( N_{Gi}^{k} )</th>
<th>( N_{Gi}^{k} )</th>
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<th>( S_{i}^{u,k} )</th>
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</tbody>
</table>
The basic step sets the price
\[ c_{gi}^q - \lambda_{gi} = 0 \quad c_{Li}^l - \lambda_{gi} = 0 \]

Repeating the same analysis for the shortage variables
\[ p^k b_{gi}^u - \lambda_{gi}^k = 0 \quad p^k b_{Li}^v - \lambda_{Li}^k = 0 \]

From the activities \( P_{gi} \) and \( L_i \)
\[ 0 = -\lambda_{gi} + \lambda_{NGi} + \sum_{k=1}^{K} \lambda_{gi}^k \]
\[ \lambda_{gi} = \lambda_{NGi} + \sum_{k=1}^{K} \lambda_{gi}^k \quad \text{Similarly} \quad \lambda_{Li} = \lambda_{NGi} + \sum_{k=1}^{K} \lambda_{Li}^k \]

The marginal value of consumption is reduced by the expected marginal losses incurred due to contingencies
\[ c_{Li}^l = \lambda_{Li} = \lambda_{NGi} + \sum_{k=1}^{K} p^k b_{Li}^v \]

or
\[ \lambda_{NGi} = \lambda_{Li} - \sum_{k=1}^{K} p^k b_{Li}^v \]

The expected marginal value of electricity is
\[ EMV_{Li} = (1 - \sum_{k=1}^{K} p^k) c_{Li}^l - \sum_{k=1}^{K} p^k b_{Li}^v \]

When \( L_i^l \) is not basic
\[ EMV_{Li} = (1 - \sum_{k=1}^{K} p^k) \lambda_{Li} - \sum_{k=1}^{K} p^k b_{Li}^v \]
The objective function with second-stage variables removed

$$\max_{P_g^l, L_l, R_l^q, S_l^q} \left( \sum_{i=1}^{N} \sum_{l=1}^{nb_i} c_{Li}^l L_l^i - \sum_{i=1}^{N} \sum_{q=1}^{no_i} c_{gi}^q P_g^q \right)$$

A Lagrangian with the removed constraints included and weighted by their duals

$$\max_{P_g^l, L_l, R_l^q, S_l^q} \left( \sum_{i=1}^{N} \sum_{l=1}^{nb_i} c_{Li}^l L_l^i - \sum_{i=1}^{N} \sum_{q=1}^{no_i} c_{gi}^q P_g^q - \left( \sum_{k \in K'} \lambda_{gi}^k \right) P_g^q - \left( \sum_{k \in K'} \lambda_{Li}^k \right) L_l^i \right)$$

$\lambda_{gi}$ and $\lambda_{Li}$ are the logical counterparts to the LMP’s in the current auction models

Comments

- Even though the first-stage prices better represent the economics of the marketplace, they still do not equate price with marginal value
- Changing prices with each contingency would add greatly to the volatility of the prices of one of the most volatile commodities
- Giving credits to consumers to account for the loss of surplus during a contingency requires a tax on consumers to create a reserve to pay for losses
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<th>Scale factor on load-loss costs</th>
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<th>0.5</th>
<th>1</th>
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<td>172</td>
<td>171</td>
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<td>169</td>
<td>169</td>
<td>169</td>
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<td>167</td>
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<td>Number of positive shortage activities</td>
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<td>14</td>
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<td>10</td>
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*Table 1: Total demand and number of positive shortage activities in the solution.*
### Table 2: Selected LMP’s in load nodes as a function of shortage costs, measured in [$/MWh], based on the load duals from constraint (3). These duals are the market-clearing prices for the day-ahead auction.

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### Table 3: Selected duals at generation nodes (LMP’s), measured in [$/MWh], based on constraint (2), as a function of shortage costs.

<table>
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<tr>
<th>Node/ Scale factor</th>
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<th>0.5</th>
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<th>2</th>
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Table 4: Dual variables that incorporate the effect of losses due to shortages and are the expected values of electricity at each node in \(\$MWh\), which are lower than the auction prices in Table 2.

<table>
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<th>Node\Scale factor</th>
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Table 5: Demand levels at selected nodes in [MWh].

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*Table 6: Rents and Surpluses for Transmission, Generation, and Demand ($)