# NODAL PRICES IN THE DAY-AHEAD MARKET

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# What we cover

- Two-stage stochastic program for contingency analysis in the day-ahead auction.
- Find the LMPs and the expected marginal value of electricity from the dual variables.
- Show differences with current duals
- Show marginal value problem
- Bouffard, Galiana, and Conejo (2005)

Objective function:

$$\max_{P_{gi}^q, L_i^l, R_i^{\nu,k}, S_i^{\nu,k}} \left( \sum_{i=1}^{N} \sum_{l=1}^{nb_i} c_{Li}^l L_i^l - \sum_{i=1}^{N} \sum_{q=1}^{no_i} c_{gi}^q P_{gi}^q \right) + \sum_{k=1}^{K} p^k \sum_{i=1}^{N} \left( \sum_{u=1}^{ns_i} b_{gi}^u S_i^{u,k} - \sum_{\nu=1}^{nr_i} b_{Li}^\nu R_i^{\nu,k} \right)$$

Inequality constraints for offer/bid blocks and total node generation/load bounds, respectively:

$$P_{gi}^{q,\min} \le P_{gi}^q \le P_{gi}^{q,\max} \qquad L_i^{l,\min} \le L_i^l \le L_i^{l,\max}$$

$$P_{gi} = \sum_{q=1}^{no_i} P_{gi}^q \quad \lambda_{gi} \qquad L_i = \sum_{l=1}^{nb_i} L_i^l \quad \lambda_{Li}$$

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max} \qquad L_i^{\min} \le L_i \le L_i^{\max}$$



Power-balance equality constraint and branch-flow inequality constraints for normal operating conditions.

$$NG_i = P_{gi} - L_i \quad \lambda_{NGi} \qquad \sum_{i=1}^{n} NG_i = 0 \quad \lambda_{bal}$$

$$-P_{bj}^{\max} \le \sum_{i=1}^{n} SF_{ji} \cdot NG_i \le P_{bj}^{\max}$$

Generation reduction and load-shedding inequality constraints, load balance equality constraints and branch flow inequality constraints for contingency conditions

$$0 \le S_i^{u,k} \le S_i^{u,k,\max} \qquad\qquad 0 \le R_i^{\nu,k} \le R_i^{\nu,k,\max}$$

$$\sum_{u=1}^{ns_i} S_i^{u,k} + P_{gi}^k = P_{gi} \qquad \lambda_{gi}^k \qquad\qquad \sum_{\nu=1}^{nr_i} R_i^{\nu,k} + L_i^k = L_i \qquad \lambda_{Li}^k$$

$$NG_i^k = P_{gi}^k - L_i^k \qquad \lambda_{NGi}^k$$

$$\sum_{i=1}^{N} NG_i^k = 0 \qquad \lambda_{bal}^k$$

$$-P_{bj}^{k,\max} \le \sum_{i=1}^{n} SF_{ji}^k \cdot NG_i^k \le P_{bj}^{k,\max} \qquad \mu_j^k$$



| Activities | $L_i^l$ | $L_i$ | $P_{gi}^q$ | $P_{gi}$ | $L_i^k$ | $P_{gi}^k$ | $NG_i$ | $NG_i^k$ | $R_i^{\nu,k}$ | $S_i^{u,k}$ | Duals |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Objective | $c_{Li}^l$ | | $-c_{gi}^q$ | | | | | | $-p^k b_{Li}^\nu$ | $p^k b_{gi}^u$ | |
| Tot load | 1 | -1 | | | | | | | | | $\lambda_{Li}$ |
| Tot gen | | | -1 | 1 | | | | | | | $\lambda_{gi}$ |
| Net gen | | 1 | | -1 | | | 1 | | | | $\lambda_{NGi}$ |
| Sys net gn | | | | | | | 1 | | | | $\lambda_{bal}$ |
| Br cap | | | | | | | $\pm SF_{ji}$ | | | | $\mu_j$ |
| Prod w/ | | | -1 | | | 1 | | | | 1 | $\lambda_{gi}^k$ |
| Dem w/ | | 1 | | 1 | -1 | | | | 1 | | $\lambda_{Li}^k$ |
| Net gen w/ | | | | | -1 | 1 | | 1 | | | $\lambda_{NGi}^k$ |
| Sys gen w/ | | | | | | | | 1 | | | $\lambda_{bal}^k$ |
| Br cap w/ | | | | | | | | $\pm SF_{ji}^k$ | | | $\mu_j^k$ |
| Bounds | $\pm 1$ | $\pm 1$ | $\pm 1$ | $\pm 1$ | $\pm 1$ | $\pm 1$ | | | 1 | 1 | |

The basic step sets the price

$$c_{gi}^{q} - \lambda_{gi} = 0 \qquad c_{Li}^{l} - \lambda_{gi} = 0$$

Repeating the same analysis for the shortage variables

$$p^{k} b_{gi}^{u} - \lambda_{gi}^{k} = 0 \qquad p^{k} b_{Li}^{v} - \lambda_{Li}^{k} = 0$$

From the activities $P_{gi}$ and $L_i$

$$0 = -\lambda_{gi} + \lambda_{NGi} + \sum_{k=1}^{K} \lambda_{gi}^{k}$$

$$\lambda_{gi} = \lambda_{NGi} + \sum_{k=1}^{K} \lambda_{gi}^{k}$$

Similarly

$$\lambda_{Li} = \lambda_{NGi} + \sum_{k=1}^{K} \lambda_{Li}^{k}$$



The marginal value of consumption is reduced by the expected marginal losses incurred due to contingencies

$$c_{Li}^{l} = \lambda_{Li} = \lambda_{NGi} + \sum_{k=1}^{K} p^{k} b_{Li}^{v}$$

or

$$\lambda_{NGi} = \lambda_{Li} - \sum_{k=1}^{K} p^{k} b_{Li}^{v}$$

The expected marginal value of electricity is

$$EMV_{Li} = (1 - \sum_{k=1}^{K} p^{k}) c_{Li}^{l} - \sum_{k=1}^{K} p^{k} b_{Li}^{v}$$

When $L_i^l$ is not basic

$$EMV_{Li} = (1 - \sum_{k=1}^{K} p^{k}) \lambda_{Li} - \sum_{k=1}^{K} p^{k} b_{Li}^{v}$$

The objective function with second-stage variables removed

$$\max_{P_{gi}^q, L_i^l, R_i^{u,k}, S_i^{v,k}} \left( \sum_{i=1}^{N} \sum_{l=1}^{nb_i} c_{Li}^l L_i^l - \sum_{i=1}^{N} \sum_{q=1}^{no_i} c_{gi}^q P_{gi}^q \right)$$

A Lagrangian with the removed constraints included and weighted by their duals

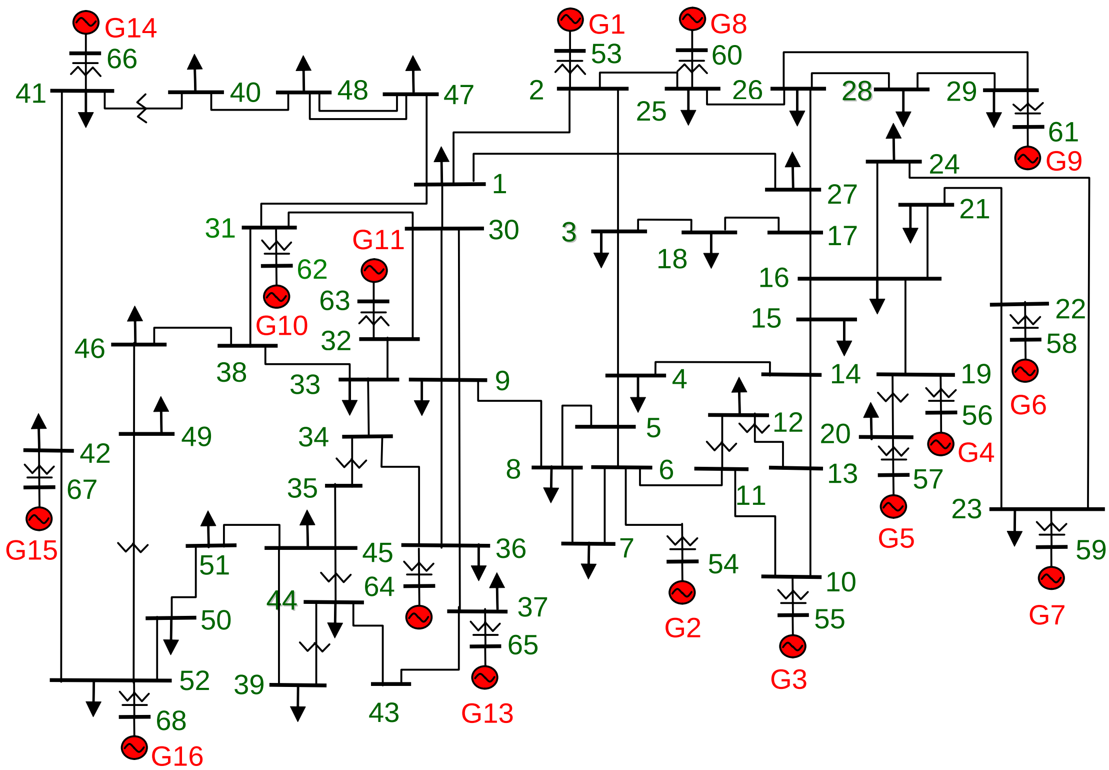
$$\max_{P_{gi}^q, L_i^l, R_i^{u,k}, S_i^{v,k}} \left( \sum_{i=1}^{N} \sum_{l=1}^{nb_i} c_{Li}^l L_i^l - \sum_{i=1}^{N} \sum_{q=1}^{no_i} c_{gi}^q P_{gi}^q - \left( \sum_{k \in K'} \lambda_{gi}^k \right) P_{gi} - \left( \sum_{k \in K'} \lambda_{Li}^k \right) L_i \right)$$

$\lambda_{gi}$ and $\lambda_{Li}$ are the logical counterparts to the LMP's in the current auction models



# Comments

- Even though the first-stage prices better represent the economics of the marketplace, they still do not equate price with marginal value
- Changing prices with each contingency would add greatly to the volatility of the prices of one of the most volatile commodities
- Giving credits to consumers to account for the loss of surplus during a contingency requires a tax on consumers to create a reserve to pay for losses






| Scale factor on load-loss costs | 0.05 | 0.2 | 0.5 | 1 | 2 | 5 | 10 | 15 | 20 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| Total demand | 172 | 172 | 171 | 170 | 169 | 169 | 169 | 169 | 168 | 167 |
| Number of positive shortage activities | 1454 | 568 | 37 | 29 | 25 | 14 | 11 | 10 | 10 | 6 |

*Table 1: Total demand and number of positive shortage activities in the solution.*

| Node\Scale factor | 0.05 | 0.2 | 0.5 | 1 | 2 | 5 | 10 | 15 | 20 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| Node 7 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| Node 16 | 54 | 54 | 55 | 55 | 55 | 54 | 54 | 54 | 54 | 54 |
| Node 18 | 56 | 56 | 56 | 56 | 55 | 54 | 52 | 52 | 50 | 48 |
| Node 20 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 |
| Node 27 | 62 | 62 | 62 | 62 | 62 | 62 | 62 | 62 | 62 | 62 |
| Node 33 | 55 | 55 | 55 | 55 | 56 | 56 | 57 | 58 | 60 | 60 |
| Node 45 | 36 | 36 | 37 | 39 | 43 | 47 | 51 | 55 | 55 | 55 |
| Node 49 | 37 | 37 | 35 | 32 | 25 | 17 | 9 | 3 | 2 | 1 |

*Table 2: Selected LMP's in load nodes as a function of shortage costs, measured in [$/MWh], based on the load duals from constraint (3). These duals are the market-clearing prices for the day-ahead auction.*



| Node\ Scale factor | 0.05 | 0.2 | 0.5 | 1 | 2 | 5 | 10 | 15 | 20 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| Node 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| Node 61 | 62 | 62 | 62 | 62 | 62 | 62 | 62 | 62 | 62 | 62 |
| Node 62 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| Node 63 | 57 | 57 | 57 | 57 | 57 | 57 | 57 | 57 | 57 | 57 |
| Node 64 | 58 | 58 | 58 | 58 | 58 | 58 | 59 | 59 | 60 | 60 |
| Node 65 | 56 | 56 | 57 | 57 | 57 | 57 | 57 | 57 | 58 | 59 |
| Node 66 | 48 | 48 | 48 | 48 | 48 | 48 | 48 | 48 | 48 | 48 |
| Node 67 | 48 | 48 | 48 | 48 | 48 | 48 | 48 | 48 | 48 | 48 |
| Node 68 | 41 | 41 | 41 | 41 | 41 | 41 | 41 | 41 | 41 | 41 |

*Table 3: Selected duals at generation nodes (LMP's), measured in [$/MWh], based on constraint (2), as a function of shortage costs.*

| Node\Scale factor | 0.05 | 0.2 | 0.5 | 1 | 2 | 5 | 10 | 15 | 20 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| Node 7 | 38. | 36 | 26. | 23 | 15 | 9 | -18 | -15 | -39 | -64 |
| Node 16 | 33 | 30 | 22 | 21 | 17 | 12 | -14 | -12 | -36 | -62 |
| Node 62 | 44 | 42 | 33 | 30 | 24 | 21 | -5 | -5 | -31 | -59 |
| Node 68 | 21 | 21 | 9 | 8 | 5 | 1 | -27 | -18 | -33 | -48 |

*Table 4: Dual variables that incorporate the effect of losses due to shortages and are the expected values of electricity at each node in [$MWh], which are lower than the auction prices in Table 2.*



| Node\Scale factor | 0.05 | 0.2 | 0.4 | 1 | 2 | 5 | 10 | 15 | 20 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| N8 | 1.17 | 1.17 | 0.54 | 0.26 | 0.11 | 0 | 0 | 0 | 0 | 0 |
| N9 | 0.57 | 0.57 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N25 | 2.71 | 2.71 | 2.71 | 2.71 | 2.71 | 2.73 | 2.80 | 2.80 | 2.80 | 2.80 |
| N37 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 0.11 | 0.11 | 0.11 |
| N40 | 2.67 | 2.67 | 2.67 | 2.67 | 2.67 | 2.67 | 2.67 | 2.67 | 2.67 | 2.61 |
| N45 | 2.22 | 2.22 | 2.22 | 1.67 | 1.40 | 1.5 | 1.60 | 1.82 | 1.81 | 1.78 |
| N46 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.04 | 1.51 | 1.04 |
| N47 | 1.51 | 1.51 | 1.51 | 1.51 | 1.51 | 1.51 | 1.42 | 1.3 | 1.11 | 0.84 |

*Table 5: Demand levels at selected nodes in [MWh].*

| Surplus\Scale factor | 0.05 | 0.2 | 0.5 | 1 | 2 | 5 | 10 | 15 | 20 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| Transmission | 96 | 89 | 88 | 79 | 74 | 86 | 103 | 103 | 86 | 81 |
| Generation | 1042 | 1037 | 1058 | 1070 | 1056 | 1048 | 1068 | 1094 | 1153 | 1173 |
| Load | 2735 | 2739 | 2718 | 2696 | 2681 | 2629 | 2538 | 2477 | 2407 | 2379 |
| Total surplus | 3873 | 3866 | 3864 | 3845 | 3813 | 3764 | 3709 | 3676 | 3647 | 3634 |

*Table 6: Rents and Surpluses for Transmission, Generation, and Demand ($)*