Modeling Risk Management in Oligopolistic Electricity Markets: A Benders Decomposition Approach

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Atlantic Energy Group, Bridging the yawning gulf between financial modeling and engineering-economic modeling for policy
Washington D.C., Sept. 17, 2008

Outline

• Goal
  – Starting Point
  – Challenge
• The model
  – Market assumptions
  – Optimization Problem of One Supplier
  – Complete Equilibrium Model
• Decomposition Approach
  – Master Problem
  – Subproblems
  – Algorithm
• Case Study
  – Algorithm improvements
• Further Research
Starting point (i)

- Market risk management & Hydrothermal coordination
  - [S.E. Fleten, 2000]
  - [G. Unger, 2002]
  - [J. Cabero, et al., 2005]
  - [A. Conejo, et al., 2008]

Optimization program of supplier g

\[ \max \quad \Pi^g \]
Subject to:
Operation constraints
Financial constraints

Starting point (ii)

- Hydrothermal coordination & Oligopolistic markets
  - [T.J. Scott, and E.G. Read, 1996]
  - [J. Bushnell, 1998]
  - [M. Rivier, et al. 2001]
  - [J. Bushnell, 2003]

Optimization program of supplier 1

\[ \max \quad \Pi^1 \]
Subject to:
Operation constraints

Optimization program of supplier g

\[ \max \quad \Pi^g \]
Subject to:
Operation constraints

Optimization program of supplier G

\[ \max \quad \Pi^G \]
Subject to:
Operation constraints

\[ s_t = S_t^0 - \alpha_t \sum_{g} q_t^g \]
Modeling Risk Management in Oligopolistic Electricity Markets: A Benders Decomposition Approach

**Challenge**

- Model and solve: Market risk management & Hydrothermal coordination & Oligopolistic markets

**Optimization program of supplier 1**

\[
\text{max } E(\Pi^1) \\
\text{Subject to:} \\
\quad \text{Operation constraints} \\
\quad \text{Financial constraints}
\]

**Optimization program of supplier g**

\[
\text{max } E(\Pi^g) \\
\text{Subject to:} \\
\quad \text{Operation constraints} \\
\quad \text{Financial constraints}
\]

\[s_t = S_t^0 - \alpha_t \cdot \sum_g q_t^g\]

**Optimization program of supplier G**

\[
\text{max } E(\Pi^G) \\
\text{Subject to:} \\
\quad \text{Operation constraints} \\
\quad \text{Financial constraints}
\]

**Market Assumptions**

- Unit operation technically feasible
- Electricity spot market
- Futures market
- CV behavior
- Electricity spot prices are variables
- Fuel spot and forward prices are data for the model

\[f_{sc,t+2,t} = E(s_{sc,t} | s_{sc,t+2})\]

Electricity Forward prices are variables
Optimization Problem of One Supplier (i)

Maximizing

Objective Function

Operation Profits +
Electricity Financial Profits −
Electricity Transaction Costs +
Fuel Financial Profits −
Fuel Transaction Costs

Subject to:

Electricity Markets

- Spot and forward price equations

Operation Constraints

- Water Reservoir Management
- Take-or-pay Contracts, etc.

Financial Constraints

- Transaction Costs

Risk

- Minimum CVaR
Optimization Problem of One Supplier (i)

Maximizing

Objective Function

Operation Profits +
Electricity Financial Profits –
Electricity Transaction Costs +
Fuel Financial Profits –
Fuel Transaction Costs

Subject to:

Minimum CVaR Risk

Electricity Markets

Operation Constraints

Financial Constraints

Risk

\[ s_{sc,t} = S_{sc,t}^0 - \alpha_t \cdot \sum_g q_{sc,t}^g \]

\[ f_{sc,l,t} = \mathbb{E} \left( s_{sc,t} \mid s_{sc,l} \right) \]

\[ \text{Transaction Costs} \]

\[ \text{Minimum CVaR} \]
Optimization Problem of One Supplier (i)

Maximizing

Objective Function

Operation Profits +
Electricity Financial Profits –
Electricity Transaction Costs +

Subject to:

Electricity Markets

Risk

Minimum CVaR
Recap of slide 6 (Challenge)

- Market risk management & Hydrothermal coordination & Oligopolistic markets

Optimization program of supplier 1

\[
\begin{align*}
\text{max } & \mathbb{E}(\Pi^1) \\
\text{Subject to:} & \text{Operation constraints} \\
& \text{Financial constraints}
\end{align*}
\]

Optimization program of supplier \(g\)

\[
\begin{align*}
\text{max } & \mathbb{E}(\Pi^g) \\
\text{Subject to:} & \text{Operation constraints} \\
& \text{Financial constraints}
\end{align*}
\]

Optimization program of supplier \(G\)

\[
\begin{align*}
\text{max } & \mathbb{E}(\Pi^G) \\
\text{Subject to:} & \text{Operation constraints} \\
& \text{Financial constraints}
\end{align*}
\]

\[
s_t = S_t^0 - \alpha_t \cdot \sum_g q_t^g
\]

Complete Equilibrium Model

- Equilibrium Model as a Linear Complementarity Problem (LCP)

KKT Optimality Conditions of Supplier 1

\[
\frac{\partial \mathcal{L}^1}{\partial q_t^1} = \frac{\partial \mathcal{L}^1}{\partial q_t^1} = 0
\]

Operation constraints

Financial Constraints

Complementary slackness

KKT Optimality Conditions of Supplier \(g\)

\[
\frac{\partial \mathcal{L}^g}{\partial q_t^g} = \frac{\partial \mathcal{L}^g}{\partial q_t^g} = 0
\]

Operation constraints

Financial Constraints

Complementary slackness

KKT Optimality Conditions of Supplier \(G\)

\[
\frac{\partial \mathcal{L}^G}{\partial q_t^G} = \frac{\partial \mathcal{L}^G}{\partial q_t^G} = 0
\]

Operation constraints

Financial Constraints

Complementary slackness

\[
s_t = S_t^0 - \alpha_t \cdot \sum_g q_t^g
\]
Decomposition Approach

- PATH is able to solve LCP with thousands of variables and constraints
- However, it is not able to solve the large-scale problem arising from:
  - Stochastic inflows, prices, demand, etc.
  - Hydrothermal coordination
  - Oligopolistic competition
  - Risk management
- Therefore, a decomposition technique is needed to deal with this kind of models
  - Equilibrium Master Problem
    - formulated as an LCP
    - solved with PATH
  - Operation Subproblems (one per supplier)
    - formulated as an LP
    - solved with CPLEX

Master Problem and Operation Subproblems

Maximizing

Objective Function

Operation Revenues – Operation Costs + Financial Profits – Transaction Costs +

Subject to:

Electricity Markets

- Spot and forward price
- Reservoir Management
- Take-or-pay Contracts
- Variable bounds

Operation Constraints

Financial Constraints

Risk

- Transaction Costs
- Minimum CVaR
Master Problem and Operation Subproblems

Maximizing

Objective Function

Operation Revenues – Operation Costs + Financial Profits – Transaction Costs +

Subject to:

Electricity Markets

➢ Spot and forward price

➢ Operation Constraints

➢ Financial Constraints

➢ Risk

Minimizing

Operation Costs

Subject to:

➢ Reservoir Management

➢ Take-or-pay Contracts

➢ Variable bounds

Minimizing

Operation Costs

Subject to:

➢ Reservoir Management

➢ Take-or-pay Contracts

➢ Variable bounds

Minimizing

Operation Costs

Subject to:

➢ Reservoir Management

➢ Take-or-pay Contracts

➢ Variable bounds

Minimizing

Operation Costs

Subject to:

➢ Reservoir Management

➢ Take-or-pay Contracts

➢ Variable bounds
Master Equilibrium Model as an LCP

KKT Optimality Conditions of Supplier 1
\[ \frac{\partial L}{\partial q_1} = \frac{\partial L}{\partial q_1} = 0 \]
Benders Cuts
Financial Constraints
Complementary slackness

KKT Optimality Conditions of Supplier g
\[ \frac{\partial L^g}{\partial q_1^g} = \frac{\partial L^g}{\partial q_1^g} = 0 \]
Benders Cuts
Financial Constraints
Complementary slackness

KKT Optimality Conditions of Supplier G
\[ \frac{\partial L^G}{\partial q_1^G} = \frac{\partial L^G}{\partial q_1^G} = 0 \]
Benders Cuts
Financial Constraints
Complementary slackness

\[ s_t = S_t^0 - \alpha_t \cdot \sum_g q_t^g \]

Equilibrium Master Problem

Algorithm

- Equilibrium Electricity prices
- Risk Hedging Contracts

Optimality Cuts
Feasibility Cuts

Suggested Productions for every supplier

Minimizing for supplier 1
Operation Costs
Subject to:
- Reservoir Management
- Take-or-pay Contracts
- Variable bounds

Minimizing for supplier g
Operation Costs
Subject to:
- Reservoir Management
- Take-or-pay Contracts
- Variable bounds

Minimizing for supplier G
Operation Costs
Subject to:
- Reservoir Management
- Take-or-pay Contracts
- Variable bounds

Optimal Production of each thermal unit
Optimal Production of each hydro unit
Case Study

- Equivalent in size to the Spanish Market
  - 12 months with 6 load levels
  - 3 generation companies
  - 80 thermal units
  - 4 equivalent hydro plants

- Uncertainty
  - 1000 scenarios of
    - Hydro inflows
    - Demand
    - Fuel costs (gas, coal and fuel oil)

- Multivariate 16-scenario tree
- 80,000 variables and constraints
Case Study: Results

Profits Supplier A

Productions Supplier C

Feasibility of subproblem A

Algorithm Improvements

- Mixed approach
  - Problems: slow convergence and solving time increases with the iterations
  - Strategy:
    1. Obtaining a good enough solution for the problem by Benders decomposition
    2. Using this solution as a starting point for the direct resolution of the problem by PATH

- Limit on the Master Problem solving time
  - If the Master Problem solving time is too long, the algorithm jumps to the next iteration before finding a solution
  - More efficient, although requires more iterations
  - Final solution is not affected by this “trick” since it is obtained by direct resolution using PATH (mixed approach)
Further Research

- Split the problem into two sequential and independent subproblems

Scenario Tree Construction

- Fuel prices
- Demand
- Hydro Inflows...

Stochastic Market Equilibrium Problem & Hydrothermal Coordination (LCP)

- Scenario tree of spot market profits: revenues - costs
- Optimal Production of each unit

Risk Management Model (LP)

- Risk Hedging Contracts

Thank you for your attention!