ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA (ICAI)
INSTITUTO DE INVESTIGACIÓN TECNOLÓGICA

Modeling Risk Management in Oligopolistic Electricity Markets: A Benders Decomposition Approach

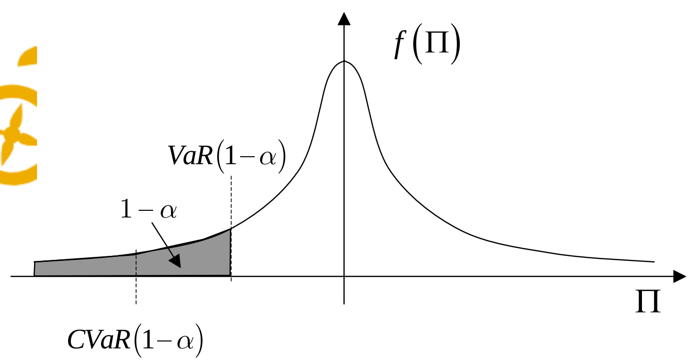
Mariano J. Ventosa
Jordi Cabero, Santiago Cerisola, Álvaro Baíllo
Atlantic Energy Group, *Bridging the yawning gulf between financial modeling and engineering-economic modeling for policy*
Washington D.C., Sept. 17, 2008

Outline

- Goal
  - Starting Point
  - Challenge
- The model
  - Market assumptions
  - Optimization Problem of One Supplier
  - Complete Equilibrium Model
- Decomposition Approach
  - Master Problem
  - Subproblems
  - Algorithm
- Case Study
  - Algorithm improvements
- Further Research

Starting point (i)

- Market risk management & Hydrothermal coordination
  - [S.E. Fleten, 2000]
  - [G. Unger, 2002]
  - [J. Cabero, *et al.*, 2005]
  - [A. Conejo, *et al.*, 2008]



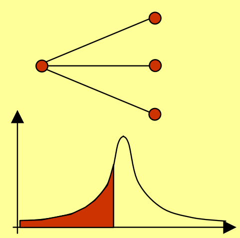
Optimization program of supplier g

$\max \ \mathbb{E}(\Pi^g)$

Subject to:

Operation constraints

Financial constraints



Starting point (ii)

- Hydrothermal coordination & Oligopolistic markets
  - [T.J. Scott, and E.G. Read,1996]
  - [J. Bushnell, 1998]
  - [M. Rivier, *et al.* 2001]
  - [J. Bushnell, 2003]

Optimization program of supplier 1	Optimization program of supplier g	Optimization program of supplier G
$\max \ \Pi^1$ Subject to: Operation constraints	$\max \ \Pi^g$ Subject to: Operation constraints	$\max \ \Pi^G$ Subject to: Operation constraints

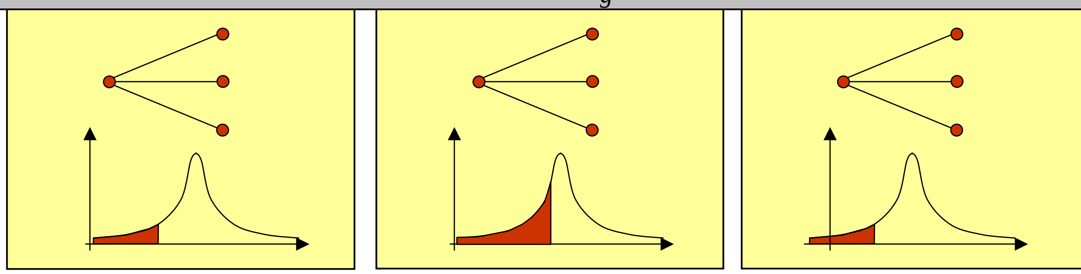
$$s_t = S_t^0 - \alpha_t \cdot \sum_g q_t^g$$

Challenge

- Model and solve: Market risk management & Hydrothermal coordination & Oligopolistic markets

Optimization program of supplier 1	Optimization program of supplier g	Optimization program of supplier G
$\max \ \mathbb{E}\left(\Pi^1\right)$ Subject to: Operation constraints, Financial constraints	$\max \ \mathbb{E}\left(\Pi^g\right)$ Subject to: Operation constraints, Financial constraints	$\max \ \mathbb{E}\left(\Pi^G\right)$ Subject to: Operation constraints, Financial constraints

$$s_t = S_t^0 - \alpha_t \cdot \sum_g q_t^g$$



Market Assumptions

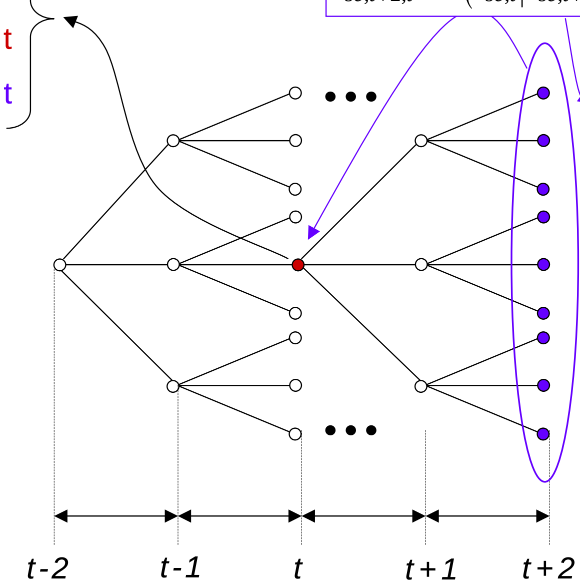
- Unit operation technically feasible
- Electricity spot market
- Futures market

- CV behavior
- Electricity spot prices are variables

Fuel spot and forward prices are data for the model

$$f_{sc,t+2,t} = \mathbb{E}\left(s_{sc,t} \middle| s_{sc,t+2}\right)$$

Electricity Forward prices are variables



Optimization Problem of One Supplier (i)

Maximizing

Objective Function → Operation Profits + Electricity Financial Profits – Electricity Transaction Costs + Fuel Financial Profits – Fuel Transaction Costs

Subject to:

Electricity Markets →
- Spot and forward price equations

Operation Constraints →
- Water Reservoir Management
- Take-or-pay Contracts, etc.

Financial Constraints →
- Transaction Costs

Risk →
- Minimum *CVaR*

Optimization Problem of One Supplier (i)

Maximizing

Objective Function

Operation Profits
Electricity Financial Profits
Electricity Transaction Costs
Fuel Financial Profits
Fuel Transaction Costs

$$\sum_{sc} P_{sc} \cdot \sum_{t} \left[\begin{array}{l} s_{sc,t} \cdot q_{sc,t} - c_{sc}\left(q_{sc,t}\right) \\ + \sum_{\substack{l \\ l<t}} \left[\left(f_{sc,l,t} - s_{sc,t} \right) \cdot \left(t^{s,e}_{sc,l,t} - t^{b,e}_{sc,l,t} \right) \right] - \sum_{\substack{k \\ k>t}} c^{e}_{sc,t,k} \\ + \sum_{\substack{l,c \\ l<t}} \left[\left(F^{c}_{sc,l,t} - S^{c}_{sc,t} \right) \cdot \left(t^{s,c}_{sc,l,t} - t^{b,c}_{sc,l,t} \right) \right] - \sum_{\substack{k,c \\ k>t}} c^{c}_{sc,t,k} \end{array} \right]$$

Optimization Problem of One Supplier (i)

Maximizing

Objective Function → Operation Profits + Electricity Financial Profits – Electricity Transaction Costs + Fuel Financial Profits – Fuel Transaction Costs

Subject to:

Electricity Mar...

$$s_{sc,t} = S^{0}_{sc,t} - \alpha_t \cdot \sum_{g} q^{g}_{sc,t} \qquad f_{sc,l,t} = \mathbb{E}\left(s_{sc,t} \middle| s_{sc,l}\right)$$

Financial Constraints → ➢ Transaction Costs

Risk → ➢ Minimum *CVaR*

Optimization Problem of One Supplier (i)

Maximizing

Objective Function → Operation Profits + ...

$$w^{j}_{sc,t} - w^{j}_{sc,t-1} - I^{j}_{sc,t} + q^{j}_{sc,t} \le 0$$

$$\left\{q_{sc,t}, \forall sc,t\right\} \in \boldsymbol{Q}, \left\{w^{j}_{sc,t}, \forall sc,t\right\} \in \boldsymbol{W}^{j}.$$

S...

Electricity Markets

Operation Constraints

Financial Constraints → ➢ Transaction Costs

Risk → ➢ Minimum *CVaR*

Optimization Problem of One Supplier (i)

Maximizing

Objective Function → Operation Profits + Electricity Financial Profits – Electricity Transaction Costs +

$$c^{e}_{sc,t,k} \ge C'^{e}_{m}\cdot\left(t^{s,e}_{sc,t,k} + t^{b,e}_{sc,t,k}\right) + C^{e}_{m}$$

$$c^{c}_{sc,t,k} \ge C'^{c}_{m}\cdot\left(t^{s,c}_{sc,t,k} + t^{b,c}_{sc,t,k}\right) + C^{c}_{m}$$

S

Electricity Markets

Operation Constraints

Financial Constraints

Risk → Minimum *CVaR*

Optimization Problem of One Supplier (i)

Maximizing

Objective Function → Operation Profits + Electricity Financial Profits – Electricity Transaction Costs + Fuel Financial Profits – Fuel Transaction Costs

Subject to:

$$z_{sc} \ge VaR_{\Pi}\left(1-\alpha\right) - \Pi_{sc}$$

$$VaR_{\Pi}\left(1-\alpha\right) - \frac{1}{1-\alpha}\cdot\sum_{sc} P_{sc}\cdot z_{sc} \ge \underline{CVaR}_{\Pi}\left(1-\alpha\right)$$

Elect
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Financi
Constraints

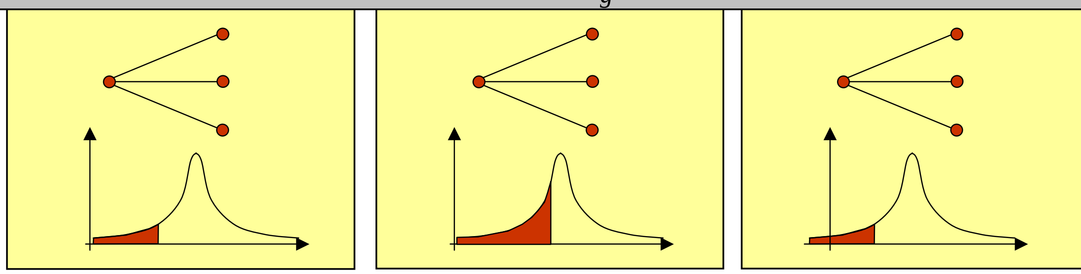
Risk

Recap of slide 6 (*Challenge*)

- Market risk management & Hydrothermal coordination & Oligopolistic markets

Optimization program of supplier 1	Optimization program of supplier g	Optimization program of supplier G
$\max\ \mathbb{E}\left(\Pi^1\right)$ Subject to: Operation constraints, Financial constraints	$\max\ \mathbb{E}\left(\Pi^g\right)$ Subject to: Operation constraints, Financial constraints	$\max\ \mathbb{E}\left(\Pi^G\right)$ Subject to: Operation constraints, Financial constraints

$$s_t = S_t^0 - \alpha_t \cdot \sum_g q_t^g$$

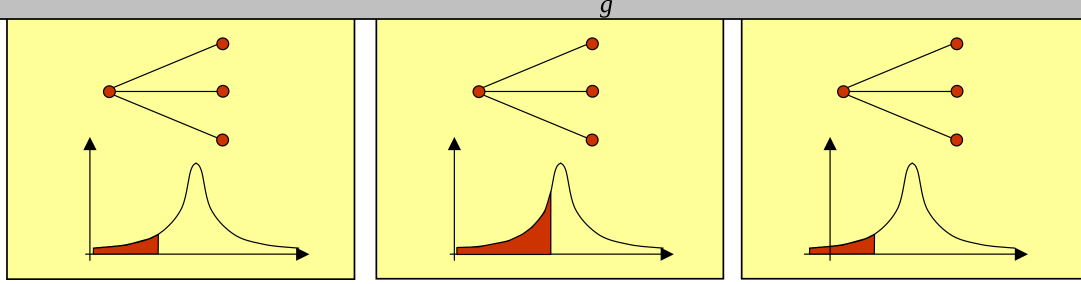


Complete Equilibrium Model

- Equilibrium Model as a Linear Complementarity Problem (LCP)

KKT Optimality Conditions of Supplier 1	KKT Optimality Conditions of Supplier g	KKT Optimality Conditions of Supplier G
$\frac{\partial \mathcal{L}^1}{\partial q_t^1} = \frac{\partial \mathcal{L}^1}{\partial q_t^1} = 0$ Operation constraints, Financial Constraints, Complementary slackness	$\frac{\partial \mathcal{L}^g}{\partial q_t^g} = \frac{\partial \mathcal{L}^g}{\partial q_t^g} = 0$ Operation constraints, Financial Constraints, Complementary slackness	$\frac{\partial \mathcal{L}^G}{\partial q_t^G} = \frac{\partial \mathcal{L}^G}{\partial q_t^G} = 0$ Operation constraints, Financial Constraints, Complementary slackness

$$s_t = S_t^0 - \alpha_t \cdot \sum_g q_t^g$$



Decomposition Approach

- PATH is able to solve LCP with thousands of variables and constraints
- However, it is not able to solve the large-scale problem arising from:
  - Stochastic inflows, prices, demand, etc.
  - Hydrothermal coordination
  - Oligopolistic competition
  - Risk management
- Therefore, a decomposition technique is needed to deal with this kind of models
  - Equilibrium Master Problem
    - formulated as an LCP
    - solved with PATH
  - Operation Subproblems (one per supplier)
    - formulated as an LP
    - solved with CPLEX

Master Problem and Operation Subproblems

Maximizing

Objective Function → Operation Revenues – Operation Costs + Financial Profits – Transaction Costs +

Subject to:

Electricity Markets →
- Spot and forward price

Operation Constraints →
- Reservoir Management
- Take-or-pay Contracts
- Variable bounds

Financial Constraints →
- Transaction Costs

Risk →
- Minimum *CVaR*

Master Problem and Operation Subproblems

Maximizing

Objective Function → Operation Revenues – Operation Costs + Financial Profits – Transaction Costs +

Subject to:

Electricity Markets → Spot and forward price

Operation Constraints → Reservoir Management; Take-or-pay Contracts; Variable bounds

Financial Constraints → Transaction Costs

Risk → Minimum *CVaR*

Minimizing

Operation Costs

Subject to:

- Reservoir Management
- Take-or-pay Contracts
- Variable bounds

Master Problem and Operation Subproblems

Maximizing

Objective Function → Operation Revenues – Op. Costs Approximation + Financial Profits – Transaction Costs +

Subject to:

Electricity Markets → Spot and forward price

Benders Cuts → **Optimality cuts**; **Feasibility cuts**

Financial Constraints → Transaction Costs

Risk → Minimum *CVaR*

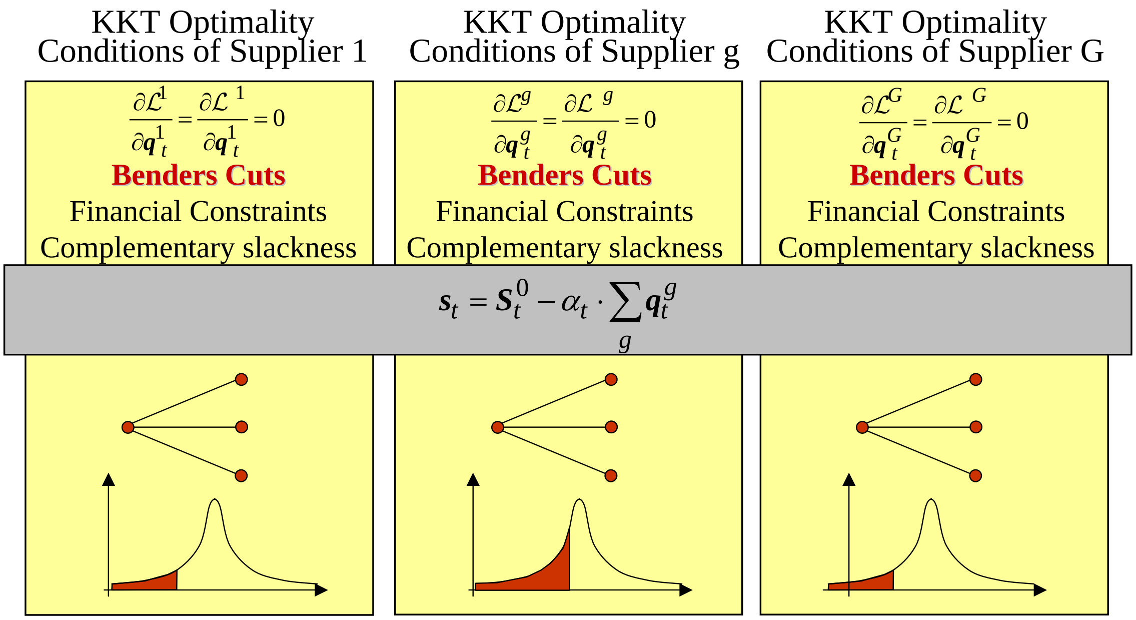
Minimizing

Operation Costs

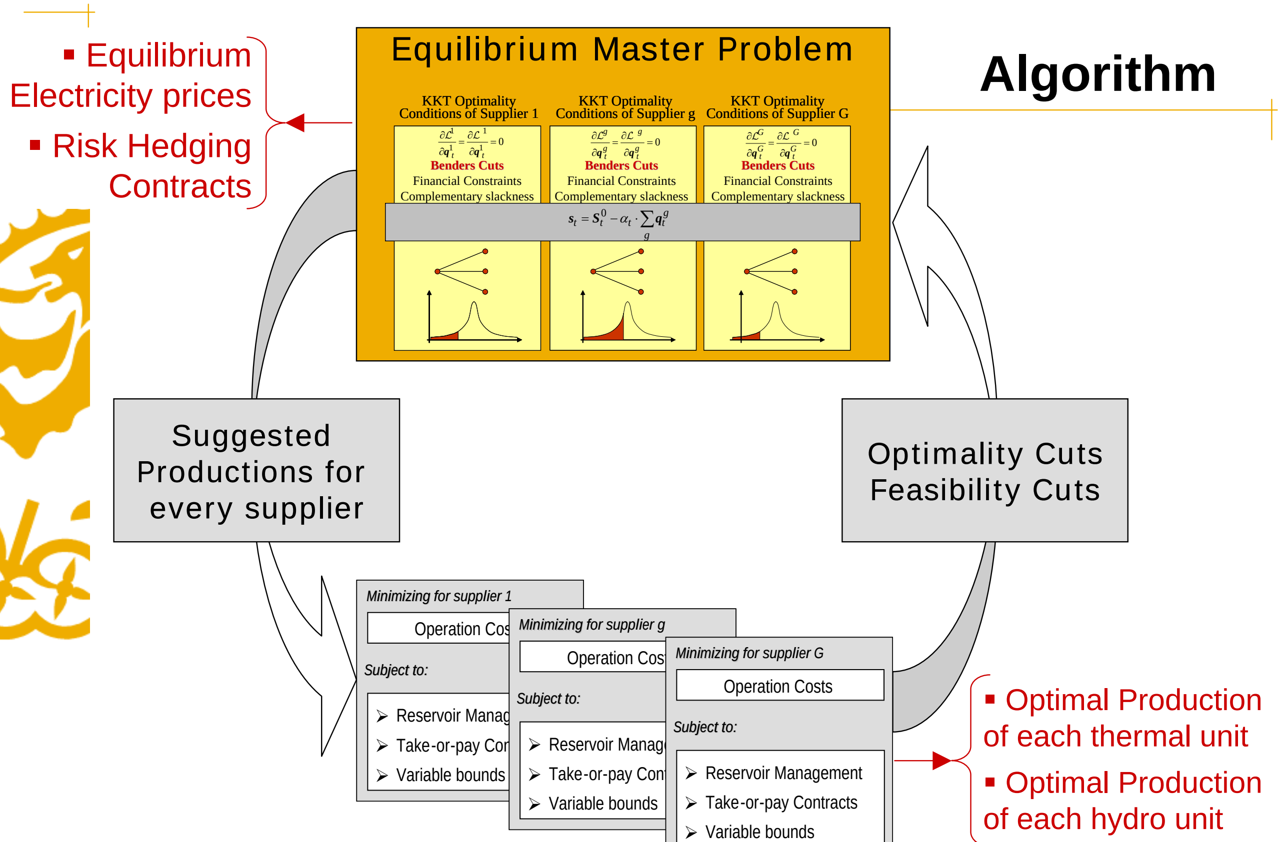
Subject to:

- Reservoir Management
- Take-or-pay Contracts
- Variable bounds

Master Equilibrium Model as an LCP



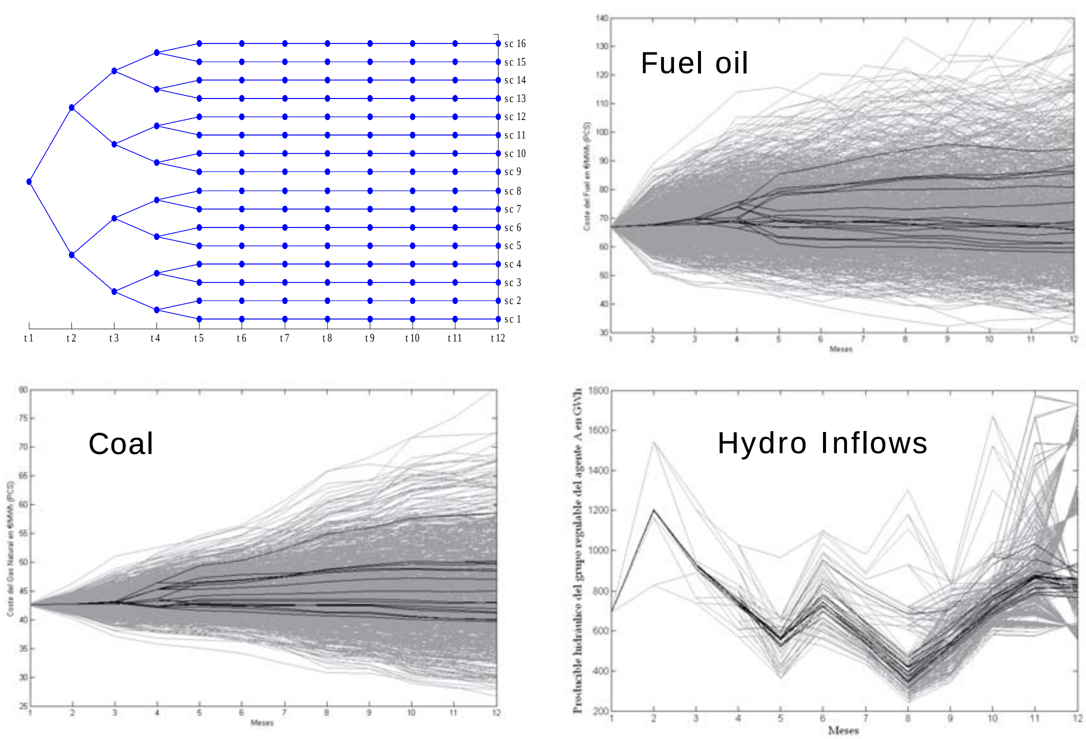
Algorithm



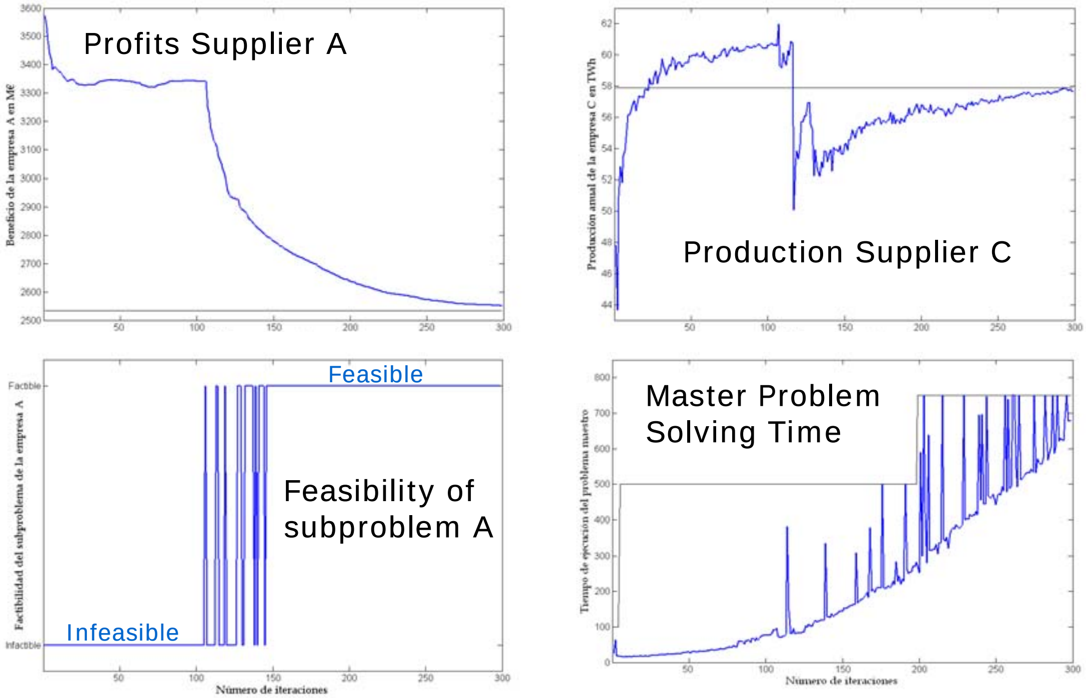
Case Study

- Equivalent in size to the Spanish Market
  - 12 months with 6 load levels
  - 3 generation companies
  - 80 thermal units
  - 4 equivalent hydro plants
- Uncertainty
  - 1000 scenarios of
    - Hydro inflows
    - Demand
    - Fuel costs (gas, coal and fuel oil)
- Multivariate 16-scenario tree
- 80,000 variables and constraints

Case Study: Scenario Tree



Case Study: Results

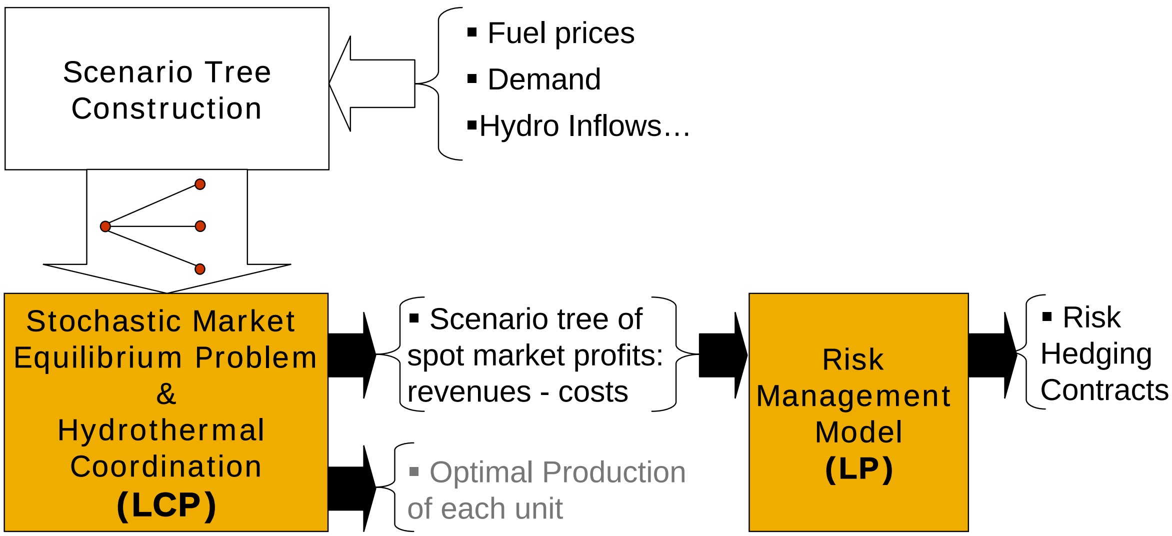


Algorithm Improvements

- Mixed approach
  - *Problems:* slow convergence and solving time increases with the iterations
  - *Strategy:*
    1. Obtaining a good enough solution for the problem by **Benders decomposition**
    2. Using this solution as a starting point for the **direct resolution** of the problem by PATH
- Limit on the Master Problem solving time
  - If the Master Problem solving time is too long, the algorithm jumps to the next iteration before finding a solution
  - More efficient, although requires more iterations
  - Final solution is not affected by this “trick” since it is obtained by direct resolution using PATH (mixed approach)

Further Research

- Split the problem into to two sequential and independent subproblems



Thank you for your attention!

Alberto Aguilera 23, E-28015 Madrid - Tel: +34 91 542 2800 - Fax: +34 91 542 2800 - http://www.iit.upcomillas.es