On the Impact of Forward Markets on Investment in Oligopolistic Markets wtih Reference to Electricity
Part 1: Deterministic Demand

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Abstract

This paper analyzes the properties of three capacity games in an oligopolistic market with Cournot players and deterministic demand. In the first game, capacity and the operation of that capacity is determined simultaneously. This is the classic open-loop Cournot game. In the second game, capacity is decided in the first stage and the operation of that capacity is determined in the second stage. The first-stage decision of each player is contingent on the solution of the second-stage game. This is a two-stage, closed-loop game. We show that when the solution exists, it is the same as the solution in the first game. However, it does not always exist. The third game has three stages with a futures position taken between the capacity stage and the operations stage and is also a closed-loop game. As with the second game, the equilibrium is the same as the open-loop game when it exists. However, the conditions for existence are more restrictive with forward markets added. When both games have an equilibrium, the solution values are identical. The results are very different from games with no capacity stage as studied by Allaz and Vila (1993), where they concluded that forward markets can ameliorate market power.

1 Introduction

One of the important questions in the theory of oligopolistic markets is the role of forward markets in mitigating market power. The literature on this subject typically examines the effect of forward markets on production levels in oligopolistic markets without explicit capacity decisions. By adding a capacity decision for each player before the decision on the forward position, we increase the realism of the game for capital-intensive industries in a commodity business and derive results that are different from those in the literature.

Understanding the effects of forward markets has taken on new importance given the problems in the California electricity market. Here the use of forward markets by the regulated electricity purchasers was restricted to 20% of expected sales and the problems in that market in 2000 were partially blamed on the lack of active forward markets that could have locked in lower rates on much of the capacity.

The original work on the potential of forward markets to mitigate market power is by Allaz (1992) and Allaz and Vila (1993). They wrote two of the early papers on this and derived the remarkable result that with Cournot players oligopolistic producers increase production just from the existence of a forward markets. This result has intuitive appeal: the forward position fixes the price for a portion of the production and reduces the quantity that is subject to lower prices from increased production. This increases the marginal revenue in the spot market for any production level, thereby increasing the equilibrium quantity. In fact, they show that as the number of periods increases, the equilibrium in a duopoly converges to the competitive equilibrium.

Their work has led to a growing literature with articles confirming or negating the result. None of these articles have addressed the effect of capacity decisions on the extent to which forward decisions can alter production decisions. In the next section we survey the literature illustrating both sides of the debate on the effect of forward markets.

Next we examine the open-loop Cournot game where capacity and production decisions are made simultaneously. We then develop closed-loop games without and with forward markets. In closed-loop games the capacity decision of each player is made knowing how this decision affects the production decision of the other player, while taking the capacity decision of the other player as given. Our first closed-loop game determines capacity in the first stage, followed by the operation of the capacity in the second stage. This is different from the standard open-loop game where the capacity and production decisions are made simultaneously and each player sees the other player's capacity and production decisions fixed. Our main results in this section are that an equilibrium might not exist, but if it exists, it is the same as the open-loop equilibrium where the capacity and production decisions are made simultaneously.

The last game has a futures stage between the capacity stage and the production stage. This is a three-stage closed-loop game. Here the capacity decisions are made knowing their effects on futures decisions, which are made knowing their effects on the spot game. The forward markets play a complex role. An equilibrium might not exist, but if it exists, it is the same as the open-loop equilibrium. This is the main result of this paper: the Allaz-Vila effect vanishes in the model with capacities, when the equilibrium exists. A further result is that there exist parameters for which the game without forward markets has an equilibrium but the game with forward markets does not. The underlying reason for these results is that capacities drastically change the way the forward market acts on the spot market. In Allaz and Villa model, increasing the forward position of a player decreases production by the other player. However, in the game with a capacity stage, both players operate at capacity with the result that increasing the forward position of a player only decreases the marginal value of the other player's capacity, not its production. We show that this property reproduces the single-stage Cournot solution.

This model is deterministic. Adding uncertainty, or the equivalent load duration curve can have an important impact as seen in Part 2.

2 Literature review

Some of the literature on the effects of forward markets on spot markets is generic to all markets, while the rest of the literature can be grouped along two different dimensions. The first dimension is Bertrand versus Cournot models with supply-function equilibria falling in between. The second is electricity markets versus traditional commodity markets.

2.1 Cournot electricity markets

Following Green and Newbery (1992), Newbery (1998) models the spot market as a supply function equilibrium. He introduces cost and demand assumptions than allow him to derive explicit supply functions. He also introduces a method for agents to coordinate on one of the different possible equilibria permitted by these functions. Given this coordination mechanism, he examines the impact of the contracts market and how contracts can have the potential to limit entry. Newbery finds the incentives to enter the contracts market differ from those of Allaz and Vila: specifically a Cournot behaviour in the contract market can in certain circumstances induce generators to avoid entering the contract market. The outcome depends on the way generators coordinate to select one equilibrium from among the infinite set of possible outcomes.

Green (1999) also models the spot market as a supply function equilibrium. He solves the problem of having a multiplicity of equilibria by restricting himself to linear supply functions. While there is an infinite set of non-linear supply function equilibria, there is a single solution in the linear case. He models competition in the contract market through different assumptions on conjectural variations and finds result that differ from Allaz Vila. Specifically the combination of Cournot behaviour in the contract market and a linear supply function equilibrium in the spot market lead generators to not enter the contract market. Green explains that this result depends on the linearity of the supply function.

In both cases these different results depend on eliminating multiple supply function equilibria using different assumptions. Gans, Price and Woods (1998) work in the context of the original model of Allaz and Vila and reaffirm their results. However, as in Newbery (1998) they note that contracts can be used to restrict entry, leading to higher prices in the long run. Their paper provides an example that demonstrates this outcome.

Harvey and Hogan (2000) start from Allaz and Vila's recognition that both players are worse off after they take their forward positions and note that the two-stage game has the payoffs of the prisoner's dilemma game with the game repeated indefinitely. It is well known that in practice, when the prisoner's dilemma game is repeated for a large number of periods, the players cooperate in the early periods. They argue that the players learn to cooperate without colluding by avoiding the futures market. A counter argument is that if the players are risk averse, they enter the forward markets for managing risk and then have the same second-stage spot game. Liski and Montero (2006) show that in the context of infinitely repeated games, forward markets reduce the gain from defection and thereby increase market power.

Using data from the beginning of the restructured markets in Australia, Wolak (2000) finds that a higher level of contracts induces increased production. He also notes that at high enough levels, contracts can lead to production levels with negative prices.

Joskow and Tirole (2002), look at the effects of the allocation of transmission rights on the electric grid. They conclude that if producers in importing regions or consumers in exporting regions own financial rights, market power is aggravated. The converse is that if producers in exporting regions and consumers in importing regions hold rights, market power is mitigated. Kamat and Oren (2004) examine the effects of zonal pricing with and without transmission constraints. They reproduce the Allaz and Vila results when the transmission constraints are not binding and show that binding constraints mitigate the effect of forward markets.

2.2 Bertrand electricity markets

Haskel and Powell (1992) show that in a contract market that is based on price offers, the market clears with price equal to marginal cost. Thus, forward markets cannot lead to increased production.

2.3 Other commodities and experiments

Le Coq and Orzen (2002) do laboratory experiments with students to test the extent to which forward markets affect spot markets. They find that a forward market leads to increased production, but not to the extent that theory would predict. Adding a forward market is not as effective as increasing the number of players because the students behaved more competitively than theory would predict.

Goering and Pippenger (2002) show that for durable commodities the optimal strategy for a monopolist is to buy in forward markets even at a premium to the spot price. This commits the monopolist to not flood the market with the durable good it produces, an example of which is metals. The commitment not to flood the market makes it possible for customers to buy more, knowing the value of their purchases will not be eroded. The monopolist has to pay a premium because the seller has the risk of being squeezed.

Mahenc and Salanie (2002) show that in a Bertrand market with partially differentiated products the optimal strategy is to take a long position in the product market. This raises the spot price and increases the profits for both players. Since prices are strategic complements, both players have an incentive to buy rather than sell futures, and, unlike Allaz and Vila, there is no prisoner's dilemma game. They note that this behavior was observed in coffee markets in 1977. They also show that the less differentiated the good, the higher the spot price.

2.4 Capacity expansion

Wu, Kleindorfer, and Sun (2002) have a capacity-expansion model in electric power with options. They did not characterize the existence of the solution or its properties with and without the options market.

Murphy and Smeers (2005) look at the capacity-expansion game as a two-stage game without a futures market in the context of electricity markets. They are able to show that the equilibrium is unique if it exists and that it does not always exist. They show that the two-stage, closed-loop formulation leads to greater capacity than an open-loop, single-period formulation. This is because each player sees the other's production decisions change in response to its increase in capacity. In some ways this is an alternative approach to imputing conjectural variation while retaining the Cournot formulation.

The next section begins with a simplified version of the model in Murphy and Smeers in that we use a deterministic demand level without a load duration curve. The model has a capacity game followed by a production game. The following sections expand on this model by adding a forward market. For uncertain demand, see Part 2.

3 Model definition

3.1 Cost structure

Assume that generation units can be entirely characterized by their investment and variable operations cost. For a given utilization rate (see Stoft (2002) for a discussion), these costs can be expressed in \$/Mwh. We let

ν_i be the per unit production cost,

k_i be the per unit capacity cost

3.2 Demand curve

We consider a single time period and let

$$p = \alpha - q \tag{1}$$

be the inverted demand curve in that period.

3.3 Variables

The most complex model considered in this paper, the three-stage closed-loop game, assumes that agents build some capacity in a first stage, trade on the forward market in a second stage and on the spot market in the third stage. Because the model is fully deterministic there is no need to distinguish between forward and futures contracts and we use these terms interchangeably. We let

x_i be the capacity invested by player i

y_i be the forward position, and

z_i be the spot generation.

This decomposition implies that y is traded in the futures market and $z - y$ in the spot market. This is the decomposition assumed in Allaz-Vila. It has different possible interpretations in electricity markets. In a standard market design interpretation, y would be traded in the day ahead market and $z - y$ in real time. In a pure bilateral system, y would correspond to OTC contracts and $z - y$ would be the trade in real time. Assuming again the most complex three-stage model, profit accruing at different stages of the decision process can be computed as follows.

Let $-i$ index the player that is not i. The profit accruing to agent i in the spot market, or third stage, is equal to

$$[\alpha - (z_i + z_{-i})](z_i - y_i) - \nu_i z_i. \tag{2}$$

By arbitrage the spot and forward prices are equal. The sale in the forward market therefore induces a revenue of $[\alpha - (z_i + z_{-i})]y_i$. There is no cost in trading forward and the forward revenue is equal to the forward profit. Thus the cumulative second- and third-stage profit in the second stage is the operating profit and equal to

$$[\alpha - (z_i + z_{-i})]z_i - \nu_i z_i. \tag{3}$$

Player i incurs a cost $-k_i x_i$ for building capacity in the first stage. This is also its first-stage profit since there is no revenue in the first stage. The cumulative three-stage profit aggregated in the first stage is thus equal to

$$[\alpha - (z_i + z_{-i})]z_i - \nu_i z_i - k_i x_i. \tag{4}$$

4 The single-stage game

The open-loop game without a forward market is the simplest of the three games considered in this paper. It is the one where agent i simultaneously decides both its investment and sales. The most natural interpretation of this game is one where both agents build capacity and immediately sell all the output of that capacity forward. There is no spot market.

With the standard Cournot assumption, the Nash equilibrium (x_i^*, x_{-i}^*) is obtained in the game when x_i^* solves

$$\max_{x_i \geq 0} \left[\alpha - (x_i + x_{-i}^*)\right] x_i - (\nu_i + k_i)x_i, \qquad i = 1, 2.$$

4.1 Equilibrium conditions

The solution to the game exists and is unique. In order to streamline the comparison of the three games (single, two, and three stages), we concentrate on the case where the single stage game has a single strictly positive equilibrium. Solving the optimization problem of each individual player, one obtains

$$\begin{aligned} &\alpha - 2x_i - x_{-i} - (\nu_i + k_i) = 0 \\ &\alpha - x_i - 2x_{-i} - (\nu_{-i} + k_{-i}) = 0. \end{aligned} \tag{5}$$

This can be solved to give

$$x_i = \frac{1}{3}\big[\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i})\big] \qquad i = 1, 2. \tag{6}$$

The price of electricity is equal to

$$\alpha - x_i - x_{-i} = \frac{1}{3}\big[\alpha + (\nu_i + k_i) + (\nu_{-i} + k_{-i})\big]. \tag{7}$$

The unit profit of player i is

$$\frac{1}{3}\big[\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i})\big] \tag{8}$$

and the total profit

$$\frac{1}{9}\big[\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i})\big]^2. \tag{9}$$

This solution has x_i strictly positive iff

$$\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i}) > 0 \qquad i = 1, 2. \tag{10}$$

The following is a trivial observation in this game.

Proposition 1 $z_i = x_i$, $i = 1, 2$ *in the open-loop game.*

5 A two-stage investment/spot model without a forward market

We now consider the case of a two-stage game, namely one where players invest in merchant plants and trade on the spot market. There is no forward market in this model. The Spanish market is an example where there is neither a bilateral nor a forward market. The equilibrium of this model is analyzed by working backward from the spot market to the capacity market.

5.1 Equilibrium conditions

Let x_i be the capacities inherited from the investment stage. The equilibrium conditions of the spot market are obtained when each agent solves the following optimization problem, which is (2), with $y_i = 0$,

$$\max_{0 \leq z_i \leq x_i} [\alpha - (z_i + z_{-i})]z_i - \nu_i z_i. \tag{11}$$

Here again, the existence and uniqueness in the equilibrium of the spot market are easily established. They are obtained as the solution of the following complementarity system.

$$\begin{aligned} &\alpha - 2z_i - z_{-i} - \nu_i + \omega_i = \lambda_i \qquad i = 1, 2 \\ &x_i - z_i \geq 0 \qquad \lambda_i \geq 0 \qquad (x_i - z_i)\lambda_i = 0 \\ &z_i \geq 0 \qquad\qquad \omega_i \geq 0 \qquad z_i\omega_i = 0 \end{aligned} \tag{12}$$

Let $z(x)$ be the solution of these equilibrium conditions as a function of the capacities x inherited from the first investment stage. It is easy to see that $z(x)$ is single valued and continuous in x. Note that $z(x)$ is not continuously differentiable in x.

In order to simplify the presentation, we limit the discussion to the case where the equilibrium satisfies $0 < z_i \leq x_i$, that is, the two producers are active at the equilibrium. Making this simplification, the equilibrium of the spot market satisfies one of the three following possible conditions.

$$\begin{aligned} &\text{(i)} \qquad 0 < z_i < x_i; \qquad i = 1, 2 \\ &\text{(ii)} \qquad 0 < z_i < x_i; \qquad 0 < z_{-i} = x_{-i} \\ &\text{(iii)} \qquad 0 < z_i = x_i; \qquad 0 < z_{-i} = x_{-i}. \end{aligned} \tag{13}$$

We now find the equilibrium in the investment game that accounts for the behavior of the players in the spot market. This is commonly referred to as a subgame-perfect equilibrium or closed-loop equilibrium (Fudenberg and Tirole (2000)). Remaining in the simple Cournot framework, we state

Definition 1 *A closed-loop equilibrium of the two-stage game $x^*, z^*(x)$ satisfies the following conditions*

(i) $z^*(x)$ is a Nash equilibrium of the spot market (second-stage game) for every feasible x

(ii) x^* is a Nash equilibrium of the capacity market game (first-stage game) where the payoffs of the agents are

$$\Pi_i(x_i; x_{-i}) = \Pi_i[x_i, z_i^*(x); x_{-i}, z_{-i}^*(x)], \qquad i = 1, 2. \tag{14}$$

If there exists a closed-loop equilibrium $x^*, z^*(x)$, then there exists a feasible neighborhood $N(x^*)$ of x^* (intersection of a ball centered on x^* and the feasible set $x \geq 0$) such that

- $z^*(x)$ is a Nash equilibrium in the spot market for all points x, $x \in N(x^*)$
- x^* is a Nash equilibrium of the capacity market with payoffs $\Pi_i(x_i, x_{-i}); i = 1, 2$, defined as in (14) in that feasible neighborhood $N(x^*)$.

$x^*, z^*(x)$ is a local equilibrium if $x^*, z^*(x)$ is an equilibrium in a feasible neighborhood around x^*. This is restated as

Definition 2 *A local closed-loop equilibrium of the two-stage game is a closed-loop equilibrium of the game where x is restricted to a non-empty full dimensional subset of the capacity space.*

We now extend Proposition 1 to the two-stage game. As a first step, we show that the cases (i) and (ii) of (13) cannot hold at equilibrium. This is done in Lemmas 1 and 2. The proofs are in the appendix.

Lemma 1 *Assume there is a closed-loop equilibrium of the two-stage game. Then case (i) of (13) cannot hold at this equilibrium.*

Lemma 2 *Suppose there exists an equilibrium of the two stage game. Then, condition (ii) of (13) cannot hold at this equilibrium.*

Proposition 2 *A closed-loop equilibrium of the two-stage game, if it exists, satisfies* $z_i^* = x_i^*, i = 1, 2$.

This immediately derives from Lemmas 1 and 2, and the assumption of existence of the closed-loop equilibrium.

Proposition 2 immediately allows us to infer the equivalence of the open and closed-loop equilibrium when the latter exists.

Theorem 1 *The closed-loop equilibrium of the two-stage game, if it exists, is identical to the open-loop equilibrium of the single-stage game.*

We now turn to the question of the existence of the equilibrium in the two-stage game. It is well known from game theory (see Fudenberg and Tirole (2000)) that existence is not guaranteed in general. An easily verifiable condition on investment cost allows one to guarantee the existence of this equilibrium for our particular problem.

Theorem 2 *Limit the capacity space* x *to points such that* $z(x) > 0$*; then there exists a closed-loop equilibrium if* $k_i \leq 2k_{-i}, i = 1, 2$.

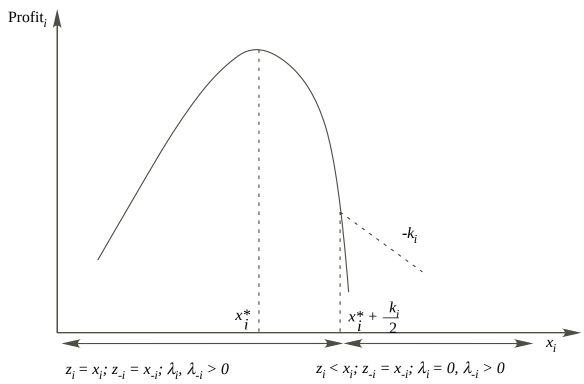

Figure 1: Profit function when $k_i < 2k_{-i}, i = 1, 2$

Figure 1 and the following discussion give the intuition of the proof. The solid curve gives the profit of player i as a function of x_i in the open loop game wen $z_{-i} = x_{-i}$. This function is concave and has a single maximand x_i^*. The profit functions of the closed and open loop games are identical when $z_i = x_i$, $i = 1, 2$. The relationships in (12) allow us to calculate the values of x_i for which $z_i = x_i$, $i = 1, 2$. At the open-loop solution x^*, $\lambda_i = k_i$, $i = 1, 2$ and we have

$$\begin{aligned} \alpha - 2x_i^* - x_{-i}^* - \nu_i &= k_i \\ \alpha - x_i^* - 2x_i^* - \nu_{-i} &= k_{-i} \end{aligned}$$

or

$$\begin{aligned} \alpha - 2(x_i^* + \tfrac{k_i}{2}) - x_{-i}^* - \nu_i &= 0 \\ \alpha - (x_i^* + k_{-i}) - x_{-i}^*) - x_{-i}^* - \nu_i &= 0. \end{aligned}$$

That is, for $x_i \leq \min(x_i^* + \frac{k_i}{2}, x_i^* + k_{-i})$, $z_i = x_i$, with $k_i < 2k_{-i}$, $z_{-i} = x_{-i}$ whenever $z_i = x_i$. At $x_i = x_i^* + \dfrac{k_i}{2}$, $\lambda_i = 0$. Thus, for $x_i > x_i^* + \dfrac{k_i}{2}$, $z_i < x_i$ and the profit function is downward sloping with slope $-k_i$ with a single maximum at x_i^*.

Consider now the case where $k_i > 2k_{-i}$. This situation is depicted in Figure 2. We now briefly explain the different segments of this curve: a more technical discussion is given in the proof in the appendix. Both λ_i and λ_{-i} are positive and hence $z_i = x_i$ and $z_{-i} = x_{-i}$ as long as $x_i < x_i^* + k_{-i}$. The profit functions of player i in the open and closed loop games are identical in that region. At $x_i = x_i^* + k_{-i}, \lambda_i > 0$ and $\lambda_{-i} = 0$. Figure 2 shows the expression of the profit function of player i for $x_i > x_i^* + k_{-i}$. Let $\widehat{x}_i$ be the maximand of that expression. In this region $z_{-i} < x_{-i}$. Three cases can occur. One can have $\widehat{x}_i < x_i^* + k_{-i}$ in which case x_i^* is the best response of player i. One can have $\widehat{x}_i \geq x_i^* + k_{-i}$ and the profit at $\widehat{x}_i$ is smaller than the one at x_i^*: x_i^* is again the optimum response. The last case is the one where $\widehat{x}_i \geq x_i^* + k_{-i}$ and the profit is higher than the one at x_i^*. $\widehat{x}_i$ is now the best response of player i. This case is analyzed in the following lemma.

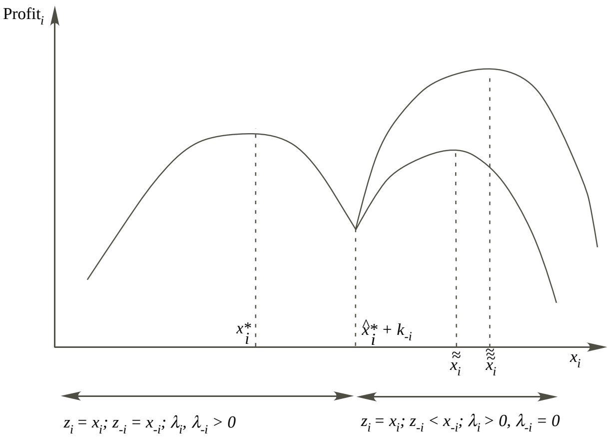

Figure 2: Profit function when $k_i > 2k_{-i}, i = 1, 2$

Lemma 3 *Suppose*

$$k_i > \frac{1}{4}\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big] > 2k_{-i},$$

then $\widehat{x}_i \geq x^*_i + k_{-i}$ *and*

$$\Pi_i(\widehat{x}_i, x^*_{-i}) = \frac{1}{8}\big[(\alpha + \nu_{-i}) - 2(\nu_i + k_i)\big]^2$$

is the profit of player i.

The proof is given in the appendix.

We now identify the condition where $\widehat{x}_i$ is an optimal response of player i to $x_{-i} = x^*_{-i}$. Recall from (9) that the profit at x^*_i, x^*_{-i} is equal to

$$\frac{1}{9}\left[\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i})\right]^2.$$

The profit of player i at $\widehat{x}_i$ is higher than at x^*_i if

$$\frac{1}{9}\big[\alpha - 2(\nu_i + k_i) + \nu_{-i} + k_{-i}\big]^2 < \frac{1}{8}\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big]^2.$$

Taking the square root of both sides we get

$$\frac{3}{2\sqrt{2}}[\alpha + \nu_{-i} - 2(k_i + \nu_i)] - \alpha + 2(\nu_i + k_i) - \nu_{-i} > k_{-i}$$

which reduces to

$$\left(\frac{3}{\sqrt{2}} - 2\right)[\alpha + \nu_{-i} - 2(k_i + \nu_i)] > 2k_{-i}. \tag{15}$$

We can then estabish the result.

Lemma 4 *If*

$$k_i > \frac{1}{4}\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big] > \left(\frac{3}{\sqrt{2}} - 2\right)\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big] > 2k_{-i} \tag{16}$$

then $\widehat{x}_i$ is the optimal response to $x_{-i} = x^*_{-i}$ when $z_i = x_i$ at $\widehat{x}_i$.

The result follows from the combination of Lemma 3 and relation (15).

We make the assumption $z_i = x_i$ in Lemma 4 because it is possible to make a different spot decision in the spot game than would seem optimal in the capacity game. The reason is that in the spot game player i sees z_i as fixed. In the capacity game player i sees how its capacity decision affects the spot equilibrium, which includes the change in z_{-i}. Since, from (12)

$$\frac{\partial z_{-i}}{\partial z_i} = -\frac{1}{2},$$

when $z_i = x_i$

$$\frac{\partial z_{-i}}{\partial x_i} = -\frac{1}{2}.$$

With this perceived response, the marginal revenue as seen in the capacity game becomes

$$\alpha - \frac{3}{2}z_i - z_{-i}$$

instead of the standard marginal revenue in the spot market of

$$\alpha - 2z_i - z_{-i}.$$

Thus, the perceived marginal revenue is higher in the capacity game with $z_i = x_i$.

The largest x for which $z_i = x_i$, which we label x_i^m, can be found by solving

$$\alpha - 2x_i - z_{-i}(x) - \nu_i = 0.$$

Replacing $z_{-i}(x)$ with its best response to x_i

$$\frac{\alpha}{2} - 3x_i + \frac{\nu_{-i}}{2} - \nu_i = 0$$

or

$$x_i^m = \frac{\alpha - \nu_{-i} - 2\nu_i}{3}$$

Say $\widehat{x}_i \geq x_i^m$, we now have to calculate the profits at x_i^m and compare the result with the profits in the open-loop solution. Here

$$z_i = x_i^m = \frac{1}{3}(\alpha + \nu_{-i} - 2\nu_i)$$

and

$$\begin{aligned} \Pi_i(x_i^m, x_{-i}^*) &= (\alpha - \tfrac{2}{3}\alpha + \tfrac{\nu_i}{3} + \tfrac{\nu_{-i}}{3} - \nu_i - k_i)\left(\tfrac{\alpha+\nu_{-i}-2\nu_i}{3}\right) \\ &= \tfrac{1}{9}(\alpha - 2\nu_i + \nu_{-i} - 3k_i)(\alpha + \nu_{-i} - 2\nu_i) \end{aligned}$$

Thus, if $\widehat{x}_i > x_i^m$, the equilibrium exists if

$$(\alpha - 2\nu_i + \nu_{-i} - 3k_i)(\alpha + \nu_{-i} - 2\nu_i) < [\alpha - 2(\nu_i + k_i) + \nu_{-i} + k_{-i}]^2.$$

This condition can be restated after some manipulation as

$$(k_i - 2k_{-i})[\alpha - 2(\nu_i + k_i) + \nu_{-i}] < 2k_i^2 + k_{-i}^2. \tag{17}$$

The following theorem summarizes the necessary and sufficient conditions for the equilibrium to exist.

Theorem 3 *Suppose we limit the capacity space x to points such that $z(x) > 0$. When $k_i > 2k_{-i}$ for some i, a closed loop equilibrium exists when the profit at $\widehat{x}_i$ is less than the profit at the equilibrium:*

$$\left(\frac{3}{\sqrt{2}} - 2\right)[\alpha + \nu_{-i} - 2(k_i + \nu_i)] < 2k_{-i}. \tag{18}$$

or when the profit at x_i^m is less than the profit at x^* and $x_i^m < \widehat{x}_i$:

$$(k_i - 2k_{-i})[\alpha - 2(\nu_i + k_i) + \nu_{-i}] < 2k_i^2 + k_{-i}^2. \tag{19}$$

A closed loop equilibrium does not exist if the profit at $\widehat{x}_i$ is greater than the profit at x^* and $\widehat{x}_i < x_i^m$:

$$k_i > \frac{1}{4}\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big] > \left(\frac{3}{\sqrt{2}} - 2\right)\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big] > 2k_{-i} \tag{20}$$

holds or $\widehat{x}_i > x_i^m$ when

$$\frac{1}{4}[\alpha + \nu_{-i} - 2(k_i + \nu_i)] > k_i$$

and the inequality (19) is reversed.

An important special case is when both players have identical costs. The properties of this game follow directly from the theorem.

Corollary 1 *When $k_i = k_{-i}$ and $\nu_i = \nu_{-i}$, the closed-loop equilibrium exists and is equal to the open-loop equilibrium.*

6 The three-stage game: the capacity game with a forward market

We now turn to the more complex case of a game where investors in merchant plants can contract part of their production in the forward market, trading the rest in the spot market. The definitions of the (local) closed-loop equilibrium of the two-stage game can be readily extended to the three-stage game after introducing additional notation. Specifically, we let z be the vector of total production in the spot market, y the amount traded forward and x the installed capacity. The three-stage game can be solved backwards as follows. A spot-price equilibrium z is a vector-valued function $z(x, y)$ where z_i solves

$$\max_{0 \le z_i \le x_i} \left\{\Pi_i^s(x, y; z_i, z_{-i}^*) = [\alpha - (z_i + z_{-i}^*)](z_i - y_i) - \nu_i z_i\right\} \qquad \text{for } i = 1, 2.$$

Assuming that this equilibrium solution exists and is unique, we can write, using (3),

$$\Pi_i^f(x;y) = \Pi_i^s[x,y;z(x;y)].$$

A forward equilibrium y is then a vector-valued point-to-set map $y(x)$ (we show that $y(x)$ need not be unique) where $y_i(x)$ solves

$$\max_{y_i} \Pi_i^f(x;y_i;y_{-i}^*) \qquad i = 1,2.$$

Assuming that this equilibrium solution exists, we define, using (4) and the above expressions

$$Z_i(x) = z_i[x;y(x)] \qquad i = 1,2.$$

We show that even though $y(x)$ is not unique, $Z_i(x)$ is unique. We can thus define

$$\Pi_i(x_i,x_{-i}) = \\ \{\alpha - [Z_i(x_i,x_{-i}) + Z_{-i}(x_i,x_{-i})] - \nu_i\} Z_i(x_i,x_{-i}) - k_i x_i \; i = 1,2.$$

The capacity equilibrium solution, if it exists, is a vector x^* where x_i^* solves

$$\max_{0 \leq x_i} \Pi_i(x_i,x_{-i}^*) \qquad i = 1,2.$$

We therefore extend Definition 1 as follows.

Definition 3 *A closed loop equilibrium of the three-stage game $x^*, y^*(x), z^*(x,y)$ satisfies the following conditions*

- *(i) $z^*(x,y)$ is a Nash equilibrium of the spot market (third-stage game) for every feasible x, y,*
- *(ii) $y^*(x)$ is a Nash equilibrium of the forward market (second stage game) for every feasible x,*
- *(iii) x^* is a Nash equilibrium of the capacity market (first stage-game).*

We now proceed to examine the different stages of this equilibrium.

6.1 The spot market equilibrium for given forward positions

Let y_i, $i = 1, 2$ be the forward position of the two agents. The equilibrium conditions on the spot market can be written as

$$\begin{array}{llll} \alpha - 2z_i - z_{-i} - \nu_i + y_i + \omega_i = \lambda_i & & & i = 1, 2 \\ (x_i - z_i) \geq 0 & \lambda_i \geq 0 & (x_i - z_i)\lambda_i = 0 & i = 1, 2 \\ z_i \geq 0 & \omega_i \geq 0 & z_i \omega_i = 0 & i = 1, 2. \end{array} \tag{21}$$

Note that y_i can be either positive or negative, corresponding to selling or buying in the futures market. Assume there exists an equilibrium $x^*, y(x^*), z[x^*; y(x^*)]$. Then the equilibrium $z^* = z[x^*, y(x^*)]$ of the spot market satisfies one of the following conditions

$$\begin{array}{lll} \text{(i)} & 0 < z_i^* < x_i^* & i = 1, 2 \\ \text{(ii)} & 0 < z_i^* < x_i^* & 0 < z_{-i}^* = x_{-i}^* \\ \text{(iii)} & 0 = z_i^* < x_i^* & 0 < z_{-i}^* < x_{-i}^* \\ \text{(iv)} & 0 = z_i^* < x_i^* & 0 < z_{-i}^* = x_{-i}^* \\ \text{(v)} & 0 < z_i^* = x_i^* & 0 < z_{-i}^* = x_{-i}^* \end{array} \tag{22}$$

We again exclude cases (iii) and (iv) where one agent is driven out of the spot market in order to shorten the discussion.

We extend Propositions 1 and 2 to the case of the three-stage game, that is, we prove that if a closed-loop equilibrium exists, it satisfies $z_i = x_i$, $i = 1, 2$ and find conditions for the existence of this equilibrium. The following lemmas are analogous to those proved in Section 5.

Lemma 5 *An equilibrium, if it exists, cannot satisfy case (i) of (22).*

The proof is given in the appendix.

We now rule out case (ii).

Lemma 6 *An equilibrium, if it exists cannot satisfy case (ii) of (22).*

The proof is also given in the appendix.

6.2 Characterization of the closed-loop equilibrium

Using these two lemmas, Proposition 3 extends Proposition 2 to the three-stage game.

Proposition 3 *A closed-loop equilibrium of the three-stage game, if it exists, satisfies* $z_i = x_i, i = 1, 2$.

With this result it is clear that the spot-market equilibrium $z_i = x_i$ is unique assuming a capacity market equilibrium.

Our next goal is to generalize Theorem 1 and to again show that if an equilibrium of the three-stage game exists, then it is the open-loop equilibrium. As with the two-stage game, we also find that this equilibrium exists only under the conditions in Theorem 1. We analyze this more complex case by partitioning the space of investment variables into different subsets where we further characterize equilibrium properties.

In the subsequent lemmas we treat cases where $z_i < x_i$. Although this cannot occur at equilibrium, this can be a property of a disequilibrium point that is relevant to showing an equilibrium does not exist. We thus have to establish the nature of the forward and spot-market equilibria for all possible $x_i > 0$.

Specifically, we first consider the case where the investment variables satisfy $\alpha - 2x_i - x_{-i} - \nu_i > 0$, $i = 1, 2$. (This is the case where both players use all of their generation capacity in the spot market). Lemma 7 characterizes the equilibrium in the forward market for that case. We then turn to the situation where one of the above inequalities is violated. This corresponds to the case where one of the players has invested in too much capacity in the sense that its marginal operating profit (marginal revenue – operating cost) on the spot market is negative when both capacities are fully used. Lemmas 8 and 9 show that the other player realizes that there is an overinvestment at the tentative equilibrium; it takes advantage of the situation and uses the forward market to drive the over-built player out of the forward market.

Lemma 7 *Let (x_i, x_{-i}) satisfy*

$$\alpha - 2x_i - x_{-i} - \nu_i > 0 \qquad i = 1, 2$$

then

$$y_i \geq \tilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) < 0, \quad i = 1, 2$$

is a closed-loop equilibrium of the forward market.

The intuition of the proof (given in the appendix) is as follows. Suppose a position $y_{-1} \geq \widetilde{y}_{-i}(x)$ of player i. Any position $y_i \geq \widetilde{y}_i(x)$ guarantees $z_i = x_i$ and $z_{-i} = x_{-i}$ and hence a constant profit of player i. A position $y_i < \widetilde{y}_i(x)$ results in $z_i < x_i$, which cannot improve the profit of player i as long as $(\alpha - 2x_i - x_{-i} - \nu_i) > 0$.

We now examine the case of overbuilding by player i. Consider now what happens when one of the relations

$$\alpha - 2x_i - x_{-i} - \nu_i > 0 \qquad i = 1, 2$$

is violated. Let

$$\alpha - 2x_i - x_{-i} - \nu_i < 0 \quad \text{and} \quad \alpha - x_i - 2x_{-i} - \nu_{-i} > 0.$$

This case corresponds to case (ii) of Theorem 2 in the game without a futures market. The analysis is more complicated with a futures market because we have to analyze the resulting futures positions of the players.

Lemma 8 shows that player $-i$ can always drive player i out of the forward market by selecting y_{-i} large enough.

Lemma 8 *For a given (x_i, x_{-i}), if $\alpha - 2x_i - x_{-i} - \nu_i < 0$ and $\alpha - x_i - 2x_{-i} - \nu_{-i} > 0$, then $y_i = 0$ is the optimal response of player i to any $y_{-i} \geq \widetilde{y}_{-i}(x)$.*

The intuition of the proof (given in the appendix) is as follows. One first shows that player $-i$ (which did not overbuild, $\alpha - x_i - 2x_{-i} - \nu_{-i} > 0)$ can always force

$z_{-i} = x_{-i}$ in the spot market whatever player i (which overbuilt $(\alpha - 2x_i - x_{-i} - \nu_i < 0)$ does, by taking any position $y_{-i} \geq \widetilde{y}_{-i}(x)$.

As shown in the proof, because $z_{-i} = x_{-i}$ and does not change with y_i, the profit function of player i in the futures market is

$$\pi_i(x, y) = \frac{1}{4}[(\alpha - x_{-i} - \nu_i)^2 - y_i^2]$$

and profit is maximized when $y_i = 0$.

Lemma 9 *Suppose $\alpha - 2x_i - x_{-i} - \nu_i < 0$ and $\alpha - x_i - 2x_{-i} - \nu_{-i} > 0$. Then $y_{-i} \geq \widetilde{y}_{-i}(x)$ is the optimal reaction of player $-i$ to $y_i = 0$.*

The idea of the proof is that the best response of player $-i$ (which did not overbuild $(\alpha - x_i - 2x_{-i} - \nu_{-i} \geq 0)$ to the strategy $y_i = 0$ of player i (which overbuilt ($\alpha - 2x_i - x_{-i} - \nu_{-i} < 0$) is to produce at capacity. It can do so by setting $y_{-i} \geq \widetilde{\widetilde{y}}_{-i}(x)$.

We now characterize the solution when player i has excess capacity.

Lemma 10 *Let (x_i, x_{-i}) satisfy $\alpha - 2x_i - x_{-i} - \nu_i < 0$ and $\alpha - x_i - 2x_{-i} - \nu_{-i} > 0$ and $\widetilde{y}_{-i}(x) = -(\alpha - x_i - 2x_{-i} - \nu_{-i})$. Then $y_i = 0$, $y_{-i} \geq \widetilde{y}_{-i}(x)$ is a closed-loop equilibrium of the forward market. At that equilibrium $z_i < x_i$.*

The result is a combination of Lemmas 8 and 9 after noting that $\widetilde{y}_{-i}(x) = -(\alpha - x_i - 2x_{-i} - \nu_{-i}) \geq -(\alpha - \widetilde{z}_i - 2x_{-i} - \nu_{-i})$.

Lemma 10 has an immediate interpretation. If a player develops its generation capacity up to a point where its marginal revenue is negative when both capacities are operated at their maximums, then the equilibrium on the forward market forces this player to operate below capacity. In short it has effectively invested too much and can increase profits by reducing capacity. We now show that this cannot be an equilibrium.

Lemma 11 *There cannot be any equilibrium of the capacity game with a forward market such that* $\alpha - 2x_i - x_{-i} - \nu_i < 0$ *and* $\alpha - x_i - 2x_{-i} - \nu_{-i} > 0$*.*

The proof is given in the appendix.

We now explore whether one can have an equilibrium of the capacity game with a forward market such that $\alpha - 2x_i - x_{-i} - \nu_i < 0$, $i = 1, 2$. This corresponds to case (ii) in the game with no forward market. The situation is easily clarified with the following lemma.

Lemma 12 *An equilibrium of the capacity game with a forward market cannot satisfy* $\alpha - 2x_i - x_{-i} - \nu_i < 0$*,* $i = 1, 2$*.*

The proof is given in the appendix.

On the basis of the above, we conclude that an equilibrium of the capacity game with a forward market, if it exists, satisfies $\alpha - 2x_i - x_{-i} - \nu_i > 0$, $i = 1, 2$ and $z_i = x_i$, $i = 1, 2$. We infer the following proposition.

Proposition 4 *An equilibrium of the capacity game with a forward market, if it exists, satisfies* $\alpha_i - 2x_i^* - x_{-i}^* - \nu_i \geq 0$*,* $i = 1, 2$*.*

Proof. The result is immediately derived from lemmas 11 and 12.

We can now present the extension of Proposition 2 to the three-stage game.

Proposition 5 *An equilibrium of the capacity game, if it exists, is the open-loop equilibrium.*

The proof is given in the appendix.

This means that the capacity game sets capacities at the same level as in the open-loop game. Thus, the futures market cannot be used to expand production in the spot market. Through the capacity game, the players see the destructive competition that results from the futures game and they block this possibility when setting capacity levels.

6.3 Existence of the closed-loop equilibrium

In the game without a futures market, existence is not guaranteed and depends on the values of the parameters. We now develop the corresponding results for the three-stage game. We take the open-loop capacities at equilibrium

$$x_i^* = \frac{1}{3}\left[\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i})\right] \quad i = 1, 2$$

and show when they are the capacity equilibrium of the three-stage game. We have a similar result as in the game with no futures market. We first note that player i never gains if it reduces its capacity with respect to x_i^*, given $x_{-i} = x_{-i}^*$. The only way the open-loop equilibrium can fail to be a first-stage equilibrium of the three-stage game is if one player can benefit from increasing its investment with respect to x_i^* with x_{-i} unchanged at x_{-i}^*. In order to explore this possibility, we increase x_i. Let $x_i = x_i^* + \varepsilon_i$ while keeping $x_{-i} = x_{-i}^*$.

Consider first the range of values of x_i that keep $\alpha - 2x_i - x_{-i}^* - \nu_i > 0$, $i = 1, 2$. We know that the equilibrium in the forward market is to select y_i, $i = 1, 2$ so that $z_i = x_i$, $i = 1, 2$. Because of the optimality properties of the open-loop equilibrium, we can conclude that player i has no interest in increasing or decreasing x_i as long as one remains in the region $\alpha - 2x_i - x_{-i} - \nu_i > 0$, $i = 1, 2$.

In order to assess whether x_i^* is really the optimal choice of player i, we need to explore what happens when a player leaves the region $\alpha - 2x_i - x_{-i} - \nu_i > 0$ for either $i = 1, 2$. Consider the two possible cases

(i) $x_i = x_i^* + \varepsilon_i$ with $\alpha - 2x_i - x_{-i}^* - \nu_i = 0$ and $\alpha - x_i - 2x_{-i}^* - \nu_{-i} > 0$

(ii) $x_i = x_i^* + \varepsilon_i$ with $\alpha - x_i - 2x_{-i}^* - \nu_{-i} = 0$ and $\alpha - 2x_i - x_{-i}^* - \nu_i > 0$

Case (i) is handled by the following lemma and is equivalent to Theorem 2.

Lemma 13 *If $k_i < 2k_{-i}$, then $x_i = x_i^*$ is the best reaction of player i to $x_{-i} = x_{-i}^*$.*

The proof (given in the appendix) essentially shows that the profit of player i as a function of x_i is identical to the one depicted in Figure 1. The reasoning is not identical though as $z_i < x_i$ for $x_i > x_i^* + \frac{k_i}{2}$ is obtained by a change of strategy on the forward market (setting $y_i = 0$). Going beyond $x_i^* + k_{-i}$ can only reduce profits by pushing player i in an overinvestment zone ($\alpha - 2x_i - x_{-i} - \nu_i < 0$).

We know from above that it is never optimal for i to select $x_i \leq x_i^* + \frac{k_i}{2}$ strictly larger than x_i^*. Consider now $x_i > x_i^* + \frac{k_i}{2}$ such that $\alpha - 2x_i - x_{-i}^* - \nu_i < 0$ and $\alpha - x_i - 2x_{-i}^* - \nu_{-i} > 0$. From Lemma 10 we know that the associated forward and spot equilibrium satisfies $y_i = 0$, $z_i < x_i$. This cannot be an optimal position for i since it can always be improved by slightly decreasing x_i.

Consider now $x_i \geq x_i^* + k_{-i}$ such that $\alpha - 2x_i - x_{-i}^* - \nu_i < 0$ and $\alpha - x_i - 2x_{-i}^* - \nu_{-i} \leq 0$. This is an optimal position for i if there exists a forward and spot equilibrium that gives a higher profit than the single-stage equilibrium. Assume such an equilibrium. It cannot satisfy $z_i = x_i$, $i = 1, 2$ because this would give a negative marginal revenue to player i even before incurring investment costs. It cannot satisfy $z_i < x_i$, $z_{-i} = x_{-i}$ because i could improve its position by decreasing x_i. It must thus satisfy $z_i = x_i$ and $z_{-i} < x_{-i}$. As in the reasoning in Lemma 8, this implies $y_{-i} = 0$ and $z_{-i} = \frac{\alpha - x_i - \nu_{-i}}{2}$. Replacing in $\alpha - 2x_i - z_{-i} - \nu_i$ the marginal revenue of i is

$$\frac{1}{2}\big[\alpha - 3x_i - (2\nu_i - \nu_{-i})\big]$$

which is equal to

$$\frac{1}{2}\big[-(\alpha - x_i - 2x_{-i}^* - \nu_{-i}) + 2(\alpha - 2x_i - x_{-i}^* - \nu_i)\big].$$

By definition, this expression is negative at $x_i = x_i^* + k_{-i}$. It can only decrease when x_i increases. The marginal revenue of player i is thus negative before incurring investment costs and this cannot be an optimal position.

Case (ii) is treated in Lemma 14 and is close to Lemma 4 in the case without futures.

Lemma 14 *Let $k_i > 2k_{-i}$ and*

$$\left(\frac{3}{\sqrt{2}} - 2\right)\left[\alpha + \nu_{-i} - 2(k_i + \nu_i)\right] - 2k_{-i} \leq 0. \tag{23}$$

Then $x_i = x_i^*$ is the best reaction of player i to $x_{-i} = x_{-i}^*$.

The proof is given in the appendix.

Consider now $x_i > x_i^* + k_{-i}$ such that $\alpha - 2x_i - x_{-i}^* - \nu_i > 0$ and $\alpha - x_i - 2x_i^* - \nu_{-i} < 0$. From Lemma 10 we know that the associated forward and spot equilibrium satisfies $y_{-i} = 0$, $z_{-i} < x_{-i}$ and that y_i is selected such that $z_i = x_i$. These conditions are analyzed in the preliminaries to Lemma 3. Because the equilibrium of the forward market guarantees that $z_i = x_i$ for $x_i \geq x_i^* + k_{-i}$, we conclude from the reasoning of Lemma 3 that the condition

$$\frac{1}{4}[\alpha + \nu_{-i} - 2(k_i + \nu_i)] > 2k_{-i}$$

implies that

$$\Pi_i(\hat{x}_i, x_{-i}^*) = \frac{1}{8}[(\alpha + \nu_{-i}) - 2(\nu_i + k_i)]^2.$$

As in Lemma 4 the assumptions guarantee that player i cannot improve its position with respect to the open loop profit by moving into a range where $z_{-i} < x_{-i}$.

Consider now $z_i \geq x_i^* + \frac{k_i}{2} > x_i^* + k_{-i}$. We then have $\alpha - 2x_i - x_{-i}^* - \nu_i < 0$ and $\alpha - x_i - 2x_{-i}^* - \nu_{-i} < 0$. These conditions have been encountered in Lemma 13 and they do not lead to an optimal position for player i.

Lemma 14 points to an interesting differentiation between the two and three stage games. In the game without a futures market, when setting capacity, player i satisfies

$$\alpha - 2x_i - z_{-i} - \nu_i \geq 0$$

as long as $x_i \leq x_i^m$. This condition guarantees that $z_i = x_i$ in the spot market.

With a futures market the condition for $z_i = x_i$ in the spot market becomes

$$\alpha - 2x_i - z_{-i} - \nu_i + y_i \geq 0. \tag{24}$$

Lemma 10 guarantees that the equilibrium of the forward market when $\alpha - 2x_i - x_{-i} - \nu_i > 0$ and $\alpha - x_i - 2x_{-i} - \nu_{-i} < 0$ is achieved for y_i large enough and $y_{-i} = 0$ and that (24) indeed holds.

We relate the effect of the futures market on the need to consider the possibility of $z_i < \widehat{x}_i$ to the Allaz-Vila result. Assume $x_i^m < \widehat{x}_i$. The optimality condition in the no-futures game is

$$\alpha - 2z_i - z_{-i} - \nu_i = 0.$$

The coefficient 2 is standard in the Cournot duopoly game. Having a futures market changes that coefficient as we now see. Initially assume $y_i = y_{-i} = 0$ and we are at the no-futures equilibrium $\widehat{z}$ in the spot market with $\widehat{z}_i < \widehat{x}_i$ and $\widehat{z}_{-i} < x_{-i}$. From (21)

$$\frac{\partial z_i}{\partial y_i} = \frac{1}{2} \text{ for } z_i < x_i$$

since

$$\frac{\partial z_{-i}}{\partial z_i} = -\frac{1}{2} \text{ for } z_{-i} < x_{-i},$$

we have

$$\frac{\partial z_{-i}}{\partial y_i} = -\frac{1}{4}.$$

The derivative of the profit function in the futures game at $\widehat{z}$ is

$$\begin{aligned} \frac{\partial \pi_i(x,y)}{\partial y} &= (\alpha - \widehat{z}_i - \widehat{z}_{-i} - \nu_i)\tfrac{1}{2} + (-\tfrac{1}{2} + \tfrac{1}{4})\widehat{z}_i \\ &= \alpha - \tfrac{3}{2}\widehat{z}_i - \widehat{z}_{-i} - \nu_i > 0 \end{aligned}$$

The inequality holds because $\frac{3}{2} < 2$. Thus, the optimal z_i in the futures optimization is greater than without a futures market. This is the Allaz-Vila effect. It turns out that adding the futures market guarantees $z_i = \widehat{x}_i$, and $x_i^m > \widehat{x}_i$. Thus the spot market equilibrium condition does not put an added condition on the existence or non existence of an equilibrium as it did in the game without a futures market.

Theorem 4 *A closed-loop equilibrium of the three-stage game exists if one of the following conditions holds*

(i) $k_i < 2k_{-i}$, $i = 1, 2$

(ii) For $k_i > 2k_{-i}$ *for some* i*, if (23) holds, then the open-loop equilibrium is also the closed-loop equilibrium of the three-stage game.*

The solution does not exist when the inequality (23) is reversed.

The proof derives from applying Lemmas 13 through 14 to both players.

From this we can see that adding a futures game does not change the equilibrium with respect to the two-stage game. However, the game with a futures market has no equilibrium for a larger set of parameter values than the game without a futures market because the condition on the spot-market equilibrium is no longer needed.

Appendix

Proof of Lemma 1

Suppose

$$0 < z_i^* < x_i^* \qquad i = 1, 2.$$

The system (12) reduces to

$$\alpha - 2z_i^* - z_{-i}^* - \nu_i = 0 \qquad i = 1, 2 \text{ or } z_i^* = \frac{1}{3}[\alpha - (2\nu_i - \nu_{-i})]$$

There exists a ball centered on x^* such that for all x in that ball, $z^*(x) = z^*$ is the best response. Therefore $(x^*, z^*(x^*))$ is a local equilibrium of the capacity game. For this equilibrium the profit of i before paying for capacity is

$$\frac{1}{9}\left[\alpha - (2\nu_i - \nu_{-i})\right]^2.$$

After paying for capacity, the profit is

$$\frac{1}{9}\big[\alpha - (2\nu_i - \nu_{-i})\big]^2 - k_i x_i^*. \quad (25)$$

However, (25) cannot be a local maximum of the payoff of player i with respect to x_i because we can reduce x_i to improve the payoff.

Proof of Lemma 2

Assume

$$0 < z_i^* < x_i^* \text{ and } z_{-i}^* = x_{-i}^*.$$

The system (12) reduces to

$$\begin{aligned} z_i^* &= \tfrac{1}{2}(\alpha - x_{-i}^* - \nu_i) \\ z_{-i}^* &= x_{-i}^* \end{aligned}$$

Set

$$\begin{aligned} z_i(x) &= \tfrac{1}{2}(\alpha - x_{-i} - \nu_i) \\ z_{-i}(x) &= x_{-i}. \end{aligned}$$

It is trivial, to verify that there exists a ball centered on x^* such that for all x in that ball $z(x)$ is the best response. Using the same argument as in Lemma 1, having $x_i^* > z_i^*$ implies that one can always decrease x_i by a small amount and achieve a higher profit. Therefore, this is not a local equilibrium and hence not an equilibrium.

Proof of Theorem 1

Let x_i^c and z_i^c, $i = 1, 2$ be the closed-loop solution of the two-stage game. By Proposition 2, this equilibrium, if it exists, satisfies $z_i^c = x_i^c$, $i = 1, 2$. This implies

$$\alpha - 2x_i^c - x_{-i}^c - \nu_i = \lambda_i^c \geq 0, \quad i = 1, 2.$$

Consider a decrease of x_i from x_i^c while keeping x_{-i} equal to x_{-i}^c. Note that as x_i^c decreases, λ_i^c increases. Thus, $z_i = x_i$, $i = 1, 2$ satisfies the spot equilibrium conditions (12). This implies that the first-stage objective function of i is equal to

$$\Pi_i(x_i; x_{-i}^c) \equiv (\alpha - x_i - x_{-i}^c - \nu_i)x_i - k_i x_i$$

when x_i is decreased with $x_{-i} = x^c_{-i}$. Note that $\Pi_i(x_i, x^c_{-i})$ is concave in x_i. Because x^c is a closed-loop equilibrium, Π_i achieves a maximum at x^c_i given $x_{-i} = x^c_{-i}$. One has

$$\alpha - 2x^c_i - x^c_{-i} - \nu_i - k_i \geq 0$$

and hence

$$\lambda^c_i \geq k_i > 0.$$

Because $\lambda^c_i > 0$, there exists a neighborhood of x^c such that for x in that neighborhood, $z_i = x_i$, $i = 1, 2$ satisfies the system (12) and hence remains the spot equilibrium in that neighborhood. Adapting the above reasoning to variations of x_i in excess of x^c_i one finds $\lambda^c_i = k_i$. Therefore, the closed-loop equilibrium of the two-stage game x^c, if it exists, satisfies the same conditions (4), as the open-loop equilibrium and hence is identical to it.

Proof of Theorem 2

Because of Theorem 1, we know that a closed-loop equilibrium of the two-stage game, if it exists, is identical to the open-loop equilibrium of the single-stage game. We, therefore, identify sufficient conditions for the open-loop equilibrium to also be a closed-loop equilibrium. The open-loop equilibrium x^* satisfies $\alpha - 2x^*_i - x^*_{-i} - \nu_i = \lambda^*_i = k_i$.

a) Let $x_i < x^*_i$. Then one can check that the second-stage equilibrium $z^*(x)$ remains $z^*(x) = x$. Because of the optimality properties of the equilibrium of the single-stage game, there cannot be a higher profit for player i when $x_i < x^*_i$.

(b) Let $x_i > x^*_i$. As x_i is increased, three possibilities can occur

(i) λ_i becomes zero before λ_{-i} becomes zero

(ii) λ_{-i} becomes zero before λ_i becomes zero.

(iii) λ_i becomes zero exactly when λ_{-i} becomes zero

We successively consider the three cases in (b).

(i) $\lambda_i = 0$ before $\lambda_{-i} = 0$. Let $\tilde{x}_i$ be the value of x_i for which λ_i reaches 0. One has

$$\alpha - 2\tilde{x}_i - x^*_{-i} - \nu_i = 0 \quad \text{or} \quad \tilde{x}_i = \tfrac{1}{2}(\alpha - x^*_{-i} - \nu_i)$$
$$\alpha - \tilde{x}_i - 2x^*_{-i} - \nu_{-i} > 0 \quad \text{or} \quad \alpha - \tfrac{1}{2}\alpha + \tfrac{1}{2}x^*_{-i} - 2x^*_{-i} + \tfrac{1}{2}\nu_i - \nu_{-i} > 0.$$

That is,

$$\frac{1}{2}\alpha - \frac{3}{2}x^*_{-i} + \frac{1}{2}(\nu_i - 2\nu_{-i}) > 0$$

or equivalently

$$\alpha + (\nu_i - 2\nu_{-i}) > 3x^*_{-i}.$$

Replacing x^*_{-i} by its equilibrium value found in the single stage game (relation (6)), we obtain that

$$\alpha + (\nu_i - 2\nu_{-i}) > \alpha - 2(\nu_{-i} + k_{-i}) + (\nu_i + k_i)$$
$$\text{or} \qquad 2k_{-i} > k_i.$$

Therefore case (i) occurs iff $2k_{-i} > k_i$. Suppose this inequality holds. One has $\alpha - 2z_i - x^*_{-i} - \nu_i = 0$ with $z_i < x_i$ for $x_i > \tilde{x}_i$. This implies that λ_{-i} never reaches 0, which in turn implies that $z_{-i} = x^*_{-i}$ and the profit Π_i is constant for $x_i > \tilde{x}_i$. Therefore, choosing $x_i > \tilde{x}_i$ cannot improve the profit of player i. Summing up, assuming $2k_{-i} > k_i$, $i = 1, 2$, neither player can increase its profit by increasing x_i with respect to the open-loop solution. This open-loop equilibrium is thus also a closed-loop equilibrium.

(ii) $\lambda_{-i} = 0$ before λ_i. Using the same steps as in (i), $k_i > 2k_{-i}$ which violates the assumption that $2k_{-i} \geq k_i$.

(iii) $x_i = \widetilde{x}_i$ simultaneously makes λ_i and λ_{-i} equal to 0. One thus has $k_i = 2k_{-i}$ and

$$\alpha - 2\widetilde{x}_i - x^*_{-i} - \nu_i = 0 \text{ and } \alpha - \widetilde{x}_i - 2x^*_{-i} - \nu_{-i} = 0. \qquad (26)$$

Let $x_i' = \widetilde{x}_i + \varepsilon, \varepsilon > 0$. $z_{-i} = x_{-i}^*$ and $z_i = \widetilde{x}_i < x_i'$ are the equilibrium in the spot market by (26). Thus,

$$\Pi_i(x_i', x_{-i}^*) = \Pi_i(\widetilde{x}_i, x_{-i}^*) - k_i\varepsilon < \Pi_i(\widetilde{x}_i, x_{-i}^*).$$

By the optimality of x_i^*, in the range $x_i^* \leq x_i \leq \widetilde{x}_i$ and the concavity of the profit function in this range

$$\Pi_i(x_i^*, x_{-i}^*) > \Pi_i(x_i, x_{-i}^*) \geq \Pi_i(\widetilde{x}_i, x_{-i}^*) > \Pi_i(x_i', x_{-i}^*)$$

which shows that it does not pay to increase x_i beyond x_i^*.

Define $\widetilde{\widetilde{x}}_i$ as the point where λ_{-i} becomes zero, as in case (b) (ii) of Theorem 2. We first note that $\widetilde{\widetilde{x}}_i$ satisfies $\alpha - \widetilde{\widetilde{x}}_i - 2x_{-i}^* - \nu_{-i} = 0$ or after substitution of the value of x_{-i}^*, $\widetilde{\widetilde{x}}_i = x_i^* + k_{-i}$. Following the reasoning of Theorem 2, the open-loop equilibrium can fail to be a closed loop equilibrium only if i has an incentive to invest x_i beyond the point $\widetilde{\widetilde{x}}_i$ where $\lambda_i > 0$ and $\lambda_{-i} = 0$. We explore this situation. Because $\lambda_{-i} = 0$, $z_{-i} < x_{-i}$ and the second-stage equilibrium implies for $x_i > \widetilde{\widetilde{x}}_i$ as long as $\lambda_i > 0$.

$$\alpha - 2x_i - z_{-i} - \nu_i = \lambda_i > 0$$
$$\alpha - x_i - 2z_{-i} - \nu_{-i} = 0$$

with $z_{-i}(x_i) = \frac{1}{2}(\alpha - x_i - \nu_{-i}) < x_{-i}^*$. Replacing z in $\Pi_i(x, z)$ by this expression while keeping $z_i = x_i$ in expression (14), the profit becomes

$$\begin{aligned} \Pi_i(x_i; x_{-i}^*)] &= \left[\alpha - x_i - \tfrac{1}{2}(\alpha - x_i - \nu_{-i}) - \nu_i\right] x_i - k_i x_i \\ &= \tfrac{1}{2}\left[\alpha - x_i + (\nu_{-i} - 2\nu_i)\right] x_i - k_i x_i. \end{aligned} \tag{27}$$

An optimum of $\Pi_i(x_i; x_{-i}^*)$ for $x_i > \widetilde{\widetilde{x}}_i$ can occur only if the derivative of $\Pi_i(x_i, x_{-i}^*)$ at $\widetilde{\widetilde{x}}_i$ is positive. Assume it is positive. What we ultimately need is that the optimal objective function value of player i for $x_i > \widetilde{\widetilde{x}}_i$ is larger than the optimum at the open-loop equilibrium. We thus compute the optimal $x_i \geq \widetilde{\widetilde{x}}_i$ that equates the derivative of $\Pi_i(x; x^*_{-i})$ to zero and verify that this optimal x_i falls in the region where (27) is a valid expression of the profit of i. Let $\widehat{x}_i$ be this value; it is equal to

$$\widehat{x}_i = \frac{1}{2}\big[\alpha + \nu_{-i} - 2\nu_i - 2k_i\big] = \frac{1}{2}\big(\alpha + \nu_{-i}\big) - (\nu_i + k_i).$$

Replacing x_i by $\widehat{x}_i$ in (27) we obtain

$$\Pi_i(\widehat{x}_i; x^*_{-i}) = \frac{1}{8}[(\alpha + \nu_{-i}) - 2(\nu_i + k_i)]^2. \tag{28}$$

In order for $\widehat{x}_i$ to be an optimal response of player i to $x_{-i} = x^*_{-i}$, we need to find the condition that guarantees that

(a) (28) is indeed the correct expression of $\Pi_i(\widehat{x}_i, x^*_{-i})$, that is, $\widehat{x}_i \geq \widetilde{\widetilde{x}}_i$ and $z_i = x_i$ when $x_i = \widehat{x}_i$.

(b) $\frac{1}{8}\big[(\alpha + \nu_{-i}) - 2(\nu_{-i} + k_i)\big]^2 > \frac{1}{9}\big[(\alpha - 2(\nu_i + k_i) + \nu_{-i} + k_{-i}\big]^2$.

We take up these two questions in the following lemma.

Proof of Lemma 3

We first find the condition for $\widehat{x}_i \geq \widetilde{\widetilde{x}}_i = x^*_i + k_{-i}$. We need

$$\frac{1}{2}\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big] - \frac{1}{3}\big[\alpha + \nu_{-i} + k_{-i} - 2(k_i + \nu_i)\big] > k_{-i}$$

or

$$\frac{1}{4}\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big] > 2k_{-i}.$$

Consider now the conditions that guarantee $z_i = x_i$ for $x_i = \widehat{x}_i$. One has $z_i = x_i$ if

$$\alpha - 2x_i - z_{-i}(x_i) - \nu_i \geq 0$$

or after replacement of $z_{-i}(x_i)$

$$\frac{\alpha}{2} - 3x_i + \frac{\nu_{-i}}{2} - \nu_i \geq 0.$$

The maximal value of x_i, x_i^m, that satisfies this condition is

$$x_i^m = \frac{\alpha + \nu_{-i} - 2\nu_i}{3}.$$

One thus needs that $\widehat{x}_i \leq x_i^m$ or

$$\frac{3}{2}(\alpha + \nu_{-i}) - 3(\nu_i + k_i) < (\alpha + \nu_{-i}) - 2\nu_i$$

that can be rewritten as

$$\frac{1}{2}(\alpha + \nu_{-i}) - \nu_{-i} - k_i < 2k_i$$

or

$$\frac{1}{4}\big[(\alpha + \nu_{-i}) - 2(k_i + \nu_i)\big] < k_i$$

which completes the lemma.

Proof of Lemma 5

Let $x^*, y^* = y(x^*), z^* = z[x^*; y(x^*)]$ be the equilibrium and assume that it satisfies condition (i) of (22). The equilibrium conditions are

$$\begin{aligned}
&\alpha - 2z_i^* - z_{-i}^* - \nu_i + y_i^* = 0 \\
&\alpha - z_i^* - 2z_{-i}^* - \nu_{-i} + y_{-i}^* = 0 \\
&0 < z_i^* < x_i^* \qquad i = 1, 2
\end{aligned}$$

Replacing $\nu_i + k_i$ by $\nu_i - y_i^*$ in the expression of the solution of the single-stage (open-loop) equilibrium (6),

$$z_i^* = z_i^*(y^*) = \frac{1}{3}[\alpha - 2(\nu_i - y_i^*) + (\nu_{-i} - y_{-i}^*)]. \tag{29}$$

There exists a neighborhood $N(y^*)$ of y^* such that (29) satisfies $0 < z_i(x^*, y) < x_i^*$ and hence remains an equilibrium of the spot market. Inserting these expressions in $\Pi_i^s[x^*, y, z]$,

$$\Pi_i^f[x^*, y] = \frac{1}{9}[\alpha - 2(\nu_i - y_i) + (\nu_{-i} - y_{-i})]^2. \tag{30}$$

Using the first order equilibrium condition,

$$y_i^* = \frac{1}{5}[\alpha - (3\nu_i - 2\nu_{-i})]$$

and $z_i^* = \frac{2}{5}[\alpha - (3\nu_i - 2\nu_{-i})]$. Thus, there exists a neighborhood $N(x^*)$ of x^* such that

$y^*(x) = y^*$ and $Z^*(x) = z^*[x, y^*(x)] = z^*$ are the best responses to any x in $N(x^*)$.

For any x in $N(x^*)$

$$\Pi_i(x) = \frac{2}{25}\left[\alpha - (3\nu_i - 2\nu_{-i})\right]^2 - k_i x_i.$$

Because $x_i^* > z_i^*$ and y_i^* does not depend on x, $\Pi_i(x)$ increases by slightly decreasing x from x_i^* which contradicts the assumption that x, $y^*(x)$, $Z^*(x)$ is an equilibrium

Proof of Lemma 6

Let $x^*, y(x^*), z[x^*, y(x^*)]$ be an equilibrium satisfying case (ii). One has

$$\begin{array}{ll} \alpha - 2z_i^* - x_{-i}^* - \nu_i + y_i^* = 0 & 0 < z_i^* < x_i^* \\ \alpha - z_i^* - 2x_{-i}^* - \nu_{-i} + y_{-i}^* = \lambda_{-i}^* & 0 < z_{-i}^* = x_{-i}^*. \end{array}$$

If it is an equilibrium, it is also a local equilibrium. Keeping x fixed at x^* and letting y move around $y(x^*)$, we find the following solution to the system

$$\begin{aligned} z_i &= \tfrac{1}{2}(\alpha - x_{-i}^* - \nu_i + y_i) \\ \lambda_{-i} &= \alpha - 2x_{-i}^* - \nu_{-i} + y_{-i} - \tfrac{1}{2}(\alpha - x_{-i}^* - \nu_i + y_i) \\ &= \tfrac{\alpha}{2} - \tfrac{3}{2}x_{-i}^* - \tfrac{1}{2}(2\nu_{-i} - \nu_i) + \tfrac{1}{2}(2y_{-i} - y_i). \end{aligned}$$

We consider the impact of a modification of y_i on the payoff of player i in the forward market given y_{-i}^* fixed. The spot price is equal to

$$\alpha - z_i - x_{-i}^* = \alpha - \frac{1}{2}(\alpha - x_{-i}^* - \nu_i + y_i) - x_{-i}^* = \frac{1}{2}(\alpha - x_{-i}^* + \nu_i - y_i).$$

The corresponding profit accruing to player i in the forward market, that is, after taking into account forward positions, is equal to

$$\begin{aligned} (\alpha - z_i - x_{-i}^* - \nu_i)z_i &= \tfrac{1}{2}(\alpha - x_{-i}^* - \nu_i - y_i)\tfrac{1}{2}(\alpha - x_{-i}^* - \nu_i + y_i) \\ &= \tfrac{1}{4}[(\alpha - x_{-i}^* - \nu_i)^2 - y_i^2]. \end{aligned}$$

By assumption, y_i^* maximizes player i's payoff for given y_{-i}^* and x^*. This implies that y_i^* must be zero. Player i's medium term payoff on the forward market is thus $\frac{1}{4}[(\alpha - x_{-i}^* - \nu_i)]^2$. This implies that the profit achieved on the capacity market is $\frac{1}{4}[(\alpha - x_{-i}^* - \nu_i)]^2 - k_i x_i^*$. Reducing x_i^* by a small amount to $x_i < x_i^*$, $y_i = 0$ remains the optimal strategy on the futures market and z_i^* remains unchanged and strictly less than x_i. This reduction improves player i's payoff in the capacity market which was therefore not optimal. This proves the lemma.

Proof of Lemma 7

Take x given and let $\widetilde{y}_i = \widetilde{y}_i(x)$, $i = 1, 2$ for this given x. One has

$$\alpha - 2x_i - x_{-i} - \nu_i + \widetilde{y}_i = 0, \quad i = 1, 2$$

and hence $z_i = x_i$ is an equilibrium on the spot market.

We want to prove that any $y_i \geq \widetilde{y}_i$ is the best response of player i to a futures position $y_{-i} \geq \widetilde{y}_{-i}$ of player $-i$. Suppose $y_i > \widetilde{y}_i$, one has

$$\begin{aligned} \alpha - 2x_i - x_{-i} - \nu_i + y_i &= \lambda_i > 0 \\ \alpha - x_i - 2x_{-i} - \nu_{-i} + y_{-i} &= \lambda_{-i} \geq 0 \end{aligned}$$

and $z_i = x_i$ remains an equilibrium on the spot market. Taking $y_i > \tilde{y}_i$ therefore maintains the profit of player i, whatever $y_{-i} \geq \widetilde{y}_{-i}$ is selected by player $-i$.
Take $y_i < \widetilde{y}_i$, $y_{-i} \geq \widetilde{y}_{-i}$. z_i becomes smaller than x_i and one can write the equilibrium conditions of the spot market as

$$\begin{aligned} \alpha - 2z_i - x_{-i} - \nu_i + y_i &= 0 \\ \alpha - z_i - 2x_{-i} - \nu_i + y_{-i} &= \lambda_{-i} > 0. \end{aligned}$$

This implies

$$z_i = \frac{1}{2}(\alpha - x_{-i} - \nu_i + y_i)$$

and

$$\Pi_i^f(x; y_i, y_{-i}) = \frac{1}{4}\big[(\alpha - x_{-i} - \nu_i)^2 - y_i^2\big].$$

The optimum of the profit of player i is achieved for $y_i = 0$ with a payoff equal to $\frac{1}{4}(\alpha - x_{-i} - \nu_i)^2$. This is the global optimum of player i if and only if

$$0 = y_i < \widetilde{y}_i = -(\alpha - 2x_i - x_{-i} - \nu_i) < 0 \quad \text{which is a contradiction.}$$

Therefore, $y_i < \widetilde{y}_i$ cannot be the best response of player i to $y_{-i} \geq \tilde{y}_{-i}$. Thus $\tilde{y}_i(x)$, $i = 1, 2$ is a closed-loop equilibrium of the forward market and any $y_i \geq \tilde{y}_i(x)$, $i = 1, 2$ is also a closed-loop equilibrium of the forward market.

Proof of Lemma 8

We first show that $z_i < x_i$ when $y_i = 0$. Suppose player $-i$ takes a position $\bar{y}_{-i} \geq \widetilde{y}_{-i}(x)$.
We first claim that the equilibrium in the spot market is

$$\begin{aligned} &\alpha - 2z_i - x_{-i} - \nu_i = 0 \\ &\alpha - z_i - 2x_{-i} - \nu_{-i} + \bar{y}_{-i} = \lambda_i \geq 0. \end{aligned}$$

To see this, first note that, because $\alpha - 2x_i - x_{-i} - \nu_i < 0$, there exists some $z_i < x_i$ (that we assume > 0) that solves $\alpha - 2z_i - x_{-i} - \nu_i = 0$. Note that the definition of $\tilde{y}_{-i}(x)$ implies $\alpha - x_i - 2x_{-i} - \nu_{-i} + \widetilde{y}_{-i}(x) = 0$ and hence any $z_i < x_i$ and $y_{-i} > \widetilde{y}_{-i}(x)$ satisfies $\alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} = \lambda_{-i} \geq 0$, which shows that $z_i < x_i$ and $z_{-i} = x_{-i}$ is the equilibrium.

Consider the reaction of player $-i$ to $y_{-i} > 0$. Because $y_{-i} \geq \widetilde{y}_{-i}(x)$, $\alpha - x_i - 2x_{-i} - \nu_{-i} + y_{-i} > 0$, $\alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} > 0$ for all $z_i < x_i$. Therefore, $z_{-i} = x_{-i}$ whenever $y_{-i} \geq \widetilde{y}_{-i}(x)$, whatever the position of player i on the forward market.

Consider the following strategies of player i, keeping in mind that $y_{-i} \geq \widetilde{y}_{-i}(x)$ implies $z_{-i} = x_{-i}$, whatever i does on the forward market. Because the shape of the objective function depends on the value of y_i, we treat two cases:

(i) $y_i \geq \widetilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) > 0$

(ii) $y_i \leq \widetilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) > 0$.

Note first that player i's payoff in case (i), remains constant at $(\alpha - x_i - x_{-i} - \nu_i)x_i$ for all $y_i \geq \widetilde{y}_i(x)$. Therefore player i cannot improve its payoff by selecting $y_i \geq \widetilde{y}_i(x)$ and the optimum in case (ii) is a global optimum.

Player i's payoff in case (ii) can be computed as follows. Because $y_i \leq \widetilde{y}_i(x)$, $z_i \leq x_i$ and z_i solves

$$\alpha - 2z_i - x_{-i} - \nu_i + y_i = 0$$
$$\alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} = \lambda_{-i} > 0.$$

As in Lemma 7, the optimal response of player i is

$$z_i = \frac{1}{2}(\alpha - x_{-i} - \nu_i + y_i) < x_i$$

and

$$\Pi_i^f(x; y_i, y_{-i}) = \frac{1}{4}\left[(\alpha - x_{-i} - \nu_i)^2 - y_i^2\right].$$

The maximum profit is achieved for $y_i = 0$ with the player i payoff equal to $\frac{1}{4}(\alpha - x_{-i} - \nu_i)^2$. This will be the global optimum of player i's payoff if one has both

$$0 = y_i < \widetilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) > 0$$

and

$$\frac{1}{4}(\alpha - x_{-i} - \nu_i)^2 > (\alpha - x_i - x_{-i} - \nu_i)x_i.$$

The first condition is true by assumption. To verify the second condition, first note that it can be rewritten

$$(\alpha - x_{-i} - \nu_i)^2 - 4(\alpha - x_{-i} - \nu_i)x_i + 4x_i^2 > 0$$

or

$$(\alpha - 2x_i - x_{-i} - \nu_i)^2 > 0$$

which is always satisfied.

The optimal reaction of player i is thus $y_i = 0$ when player $-i$ selects $y_{-i} \geq \bar{y}_{-i}$ and $\alpha - 2x_i - x_{-i} - \nu_i < 0$. Note that this solution is unique by the strict concavity of the objective function in this range. This proves the lemma.

Proof of Lemma 9

With $y_i = 0$, define $\widetilde{z}_i$ such that $\alpha - 2\widetilde{z}_i - x_{-i} - \nu_i = 0$. Because $\alpha - 2x_i - x_{-i} - \nu_i < 0$, $\widetilde{z}_i$ is smaller than x_i. We consider three cases because the shape of the objective function of player $-i$ depends on whether the spot decisions are at capacity. We examine the following strategies of player $-i$ on the forward market.

(i) y_{-i} is selected to guarantee $z_{-i} = x_{-i}$.

(ii) y_{-i} is selected to optimize the payoff in the range where $z_i < x_i$, $z_{-i} < x_{-i}$.

(iii) y_{-i} is selected to optimize the payoff in the range where $z_i = x_i$, $z_{-i} < x_{-i}$.

We successively consider these three cases and compute the resulting payoff for player $-i$.

(i) Player $-i$ uses the futures market to guarantee the full utilization of its capacity and it takes $y_{-i} \geq \widetilde{\widetilde{y}}_{-i}(x)$ where $\widetilde{\widetilde{y}}_{-i}(x)$ is defined by

$$\alpha - \widetilde{z}_i - 2x_{-i} - \nu_{-i} + \widetilde{\widetilde{y}}_{-i}(x) = 0.$$

This amounts to selecting

$$y_{-i} \geq \widetilde{\widetilde{y}}_{-i}(x).$$

The equilibrium on the spot market associated with $y_i = 0$, $y_{-i} \geq \widetilde{\widetilde{y}}_{-i}(x)$ is $z_i = \widetilde{z}_i$ and $z_{-i} = x_{-i}$. The payoff for player $-i$ is

$$(\alpha - \widetilde{z}_i - x_{-i} - \nu_i)x_{-i} = \frac{1}{2}(\alpha - x_{-i} - \nu_i)x_i.$$

(ii) Let $y_{-i} = \widetilde{\widetilde{y}}_{-i}(x) - \varepsilon_{-i}$ where ε_{-i} is small enough to guarantee that z_i does reach x_i and z_{-i} does not hit zero. z_i and z_{-i} then solve the system

$$\alpha - 2z_i - z_{-i} - \nu_i = 0$$
$$\alpha - z_i - 2z_{-i} - \nu_{-i} + y_{-i} = 0$$

We can solve for z_i and z_{-i} as a function of y_{-i}, as in Lemma 5. Setting $y_i = 0$ in relation (30), the payoff for $-i$ is

$$\Pi^f_{-i}[x; 0, y_{-i}] = \frac{1}{9}\left[\alpha - 2(\nu_{-i} - y_{-i}) + \nu_i\right]^2.$$

The derivative of Π^f_{-i} with respect to y_{-i} is $\frac{4}{9}[\alpha - 2(\nu_{-i} - y_{-i}) + \nu_i]$. At $\tilde{\tilde{y}}_{-i}$, when z_{-i} reaches x_{-i}, this derivative is equal to

$$\frac{4}{9}\left[\alpha - (2\nu_{-i} + \nu_i) + 2\tilde{\tilde{y}}_{-i}\right] = \frac{4}{9}x_{-i} > 0.$$

Because $\Pi^f_{-i}[x; 0, y_{-i}]$ is concave in y_{-i}, and its derivative at $\tilde{\tilde{y}}_{-i}(x)$ is positive, it is still increasing at that point. Thus, the optimum of $\Pi^f_{-i}[x; 0, y_{-i}]$ cannot be $y_{-i} < \tilde{\tilde{y}}_{-i}(x)$. This implies that $y_{-i} = \tilde{\tilde{y}}_{-i} - \varepsilon_{-i}$ is not the best response by player $-i$.

(iii) The following elaborates on the same concavity argument to prove that decreasing y_{-i} to the level where z_i reaches x_i or z_{-i} reaches 0 cannot maximize $\Pi^f_{-i}[x; 0, y_{-i}]$. There is obviously no gain for player $-i$ to further decrease y_{-i} if z_{-i} hits zero before z_i reaches x_i since its payoff is then exactly zero. Consider the alternative case where z_i hits x_i and z_{-i} is still positive. This occurs for some $\overline{z}_{-i}$ that satisfies

$$\alpha - 2x_i - \overline{z}_{-i} - \nu_i = 0$$
$$\text{or} \qquad \overline{z}_{-i} = \alpha - 2x_i - \nu_i.$$

Consider decreasing y_{-i} further to check the possibility of the resulting price increasing profits. We show that this cannot happen. Let $z_{-i} = \overline{z}_{-i} + \varepsilon$. The corresponding profit of player $-i$ is

$$(\alpha - x_i - \overline{z}_{-i} - \varepsilon - \nu_{-i})(\overline{z}_{-i} + \varepsilon).$$

The derivative of this expression at $\varepsilon = 0$ (for $z_{-i} = \overline{z}_{-i}$) is equal to $3x_i + (2\nu_i - \nu_{-i}) - \alpha$. This expression is positive because it is equal to

$$2(-\alpha + 2x_i + x_{-i} + \nu_i) + (\alpha_i - x_i - 2x_{-i} - \nu_{-i})$$

which is positive by assumption.

The conclusion is that it cannot pay to further decrease y_i beyond the point where $z_i = x_i$. $y_{-i} \geq \widetilde{y}_{-i}(x)$ thus guarantees the maximal profit of player $-i$ when $y_i = 0$.

This completes the proof of the lemma.

Proof of Lemma 11

Assume such an equilibrium exists. The equilibrium on the forward market is $y_i = 0$ and $y_{-i} \geq \bar{y}_{-i}$ with the corresponding spot equilibrium $z_i = \frac{1}{2}(\alpha - x_{-i} - \nu_i)$, $z_{-i} = x_{-i}$. This spot equilibrium satisfies $z_i < x_i$ and hence cannot be an equilibrium by Proposition 3.

Proof of Lemma 12

If such an equilibrium exists, it satisfies $z_i = x_i$, $i = 1, 2$ by Proposition 3. Because the marginal revenue of both player is negative at this point, this cannot be an optimal position for either of them. Therefore, this is not an equilibrium.

Proof of Proposition 5

Assume an equilibrium of the three-stage game exists. By Proposition 4, one has $\alpha - 2x_i - x_{-i} - \nu_i \geq 0$, $i = 1, 2$. $\alpha - 2x_i - x_{-i} - \nu_i$ is also the marginal operating profit accruing to player i from its operation on the forward and spot market (both players select y_i such that $z_i = x_i$). The optimality of player i's action in the capacity game implies that the marginal operating profit is equal to k_i. We therefore need $\alpha - 2x_i - x_{-i} - \nu_i - k_i = 0$, $i = 1, 2$. These are the conditions for the open-loop equilibrium. We thus conclude that if an equilibrium of the three-stage game exists, it is the open-loop equilibrium.

Proof of Lemma 13

Set $x_i = x_i^* + \varepsilon_i$. Simple replacement in $\alpha - 2x_i - x_{-i}^* - \nu_i = 0$ and $\alpha - x_i - 2x_i^* - \nu_{-i} > 0$ shows that case (i) holds if and only if $2k_{-i} > k_i$. If so $\alpha - 2x_i - x_{-i}^* - \nu_i = 0$ for $\varepsilon_i = \frac{k_i}{2}$.

Proof of Lemma 14

Following the reasoning of Lemma 12, we can easily verify that case (ii) occurs if and only if $k_i > 2k_{-i}$. We know that selecting x_i such that $x_i^* < x_i \leq x_i^* + k_{-i}$ would imply $z_i = x_i$, $z_{-i} = x_{-i}^*$ which cannot be an optimal payoff of player i in that range of x_i.

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